Minimum Wages in the UK: Searching for Nonlinearities

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Abstract

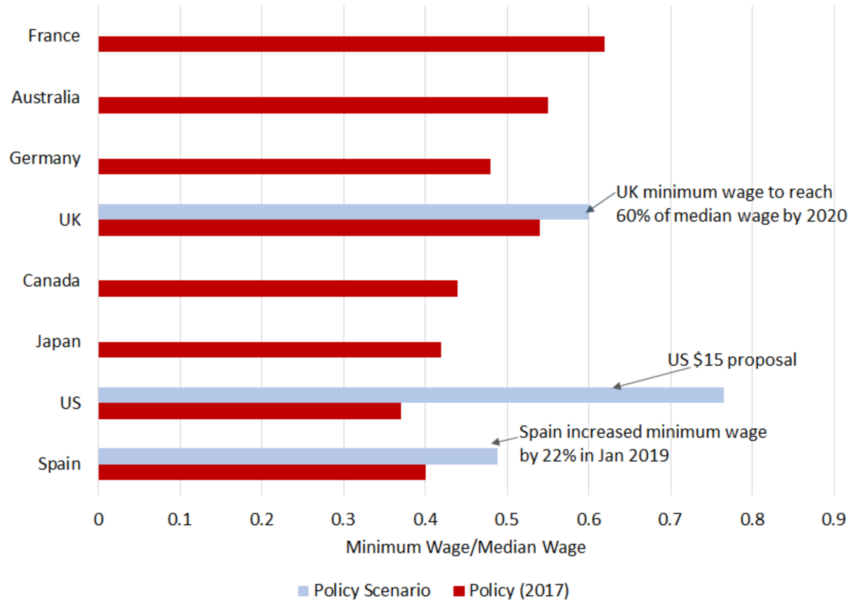
This paper examines the impact of minimum wages when search frictions are present and firms can substitute away from low skilled workers to both higher skilled workers and to capital. This represents a contribution to the search literature, which typically assumes labour is the only input of production and perfect substitution between labour inputs. I examine whether the model I develop features significant nonlinearities in the impact of the minimum wage on unemployment. I find that the theoretical contribution of this paper, i.e. allowing for search frictions and imperfect substitutability of factor inputs, is quantitatively significant. Specifically, the nonlinear unemployment response in my model is much less pronounced if I use the typical assumptions of the search literature, which imply a considerably more linear response of unemployment to the minimum wage.

Keywords: Search Frictions, Minimum Wages, Monopsony, Labor Markets

1. Introduction

Minimum wages are an increasingly popular policy response to concerns about low pay growth and wage inequality. In the 2015 Budget the former UK Chancellor announced a significant increase in the minimum wage, taking it from around 45% of the median wage in 2015 to a planned level of 60% of the median wage in 2020. UK policy makers are not the only ones turning towards higher minimum wages, as shown in Figure 1. In the US, there is an active campaign to increase the minimum wage to $15, which has had considerable success at a state/municipality level if not at the federal level. The German government introduced a $11.75 minimum wage in 2015, where previously trade unions had been the sole form of protection against low pay. More recently, the Spanish Government increased the minimum wage by 22% as of January 2019.

Figure 1. Minimum Wages on the Rise



Source: OECD, own calculations

Much of the academic literature has focused on econometric evaluation of past increases to the minimum wage, and particularly on estimating impacts on employment rates. In the UK at least, the consensus of this empirical literature seems to be that the introduction of the minimum wage in 1998, and subsequent increases in the 2000s, had relatively benign effects: increasing wages for low paid workers without a significant decrease in employment (Draca et al. (2011), Stewart

(2004)). This has led many to call for further increases; for example the Labour party proposed a £10 minimum wage (current rate is £8.21 for employees aged 25 and over) in their 2017 election manifesto. This represents something of a risk as past performance may not be a reliable guide to future impacts.

This paper explores just how risky this logic could be by examining, in the context of the UK economy: (i) whether there are likely to be significant nonlinearities in the impact of the minimum wage on unemployment; and (ii) what channels drive the location and severity of any such nonlinearities.

Given this involves considering minimum wage levels outside of those already observed in the UK, any answer will require a structural model of the economy. I consider a reasonable requirement of any such model is that it replicates the impacts of the introduction of the minimum wage described above. Equally, it should also be able to asses the risk of less favourable impacts as the minimum wage is increased to significantly higher levels. I therefore propose a model that combines a production process featuring several margins of substitution between factor inputs with frictional labour markets.

Frictional labour markets, and the monopsony power they imply, are likely to be necessary to replicate the results of empirical studies in the UK concerning the introduction of the minimum wage. In particular, findings of a significant increase in wages, a fall in corporate profits, without an increase in firm exits (Draca et al. (2011)), are suggestive of some degree of monopsony power, as is the absence of significant unemployment impacts (Leonard et al. (2014)). This consideration also points towards a wage bargaining model with random search, rather than a directed search or wage posting model, which tend to replicate many of the features of competitive labour market models.

I model the labour market using a heterogeneous agent on-the-job (OTJ) search model, with a similar wage bargaining mechanism to Cahuc et al. (2006). I consider heterogeneity a necessary ingredient as the biting point of the minimum wage on employment is likely to depend on the ability distribution of workers. OTJ search provides an endogenous source of worker bargaining power and employer competition that can, to some extent, be disciplined by the data.

To this labour market structure, I add two features that are potentially helpful in analysing the employment reaction to minimum wages and the latter of which represents a key contribution of my approach: (i) endogenous vacancy creation; and (ii) firms that can respond to minimum wage increases by substituting both capital and high skilled labour for low skilled labour using the production function developed and estimated in Krusell et al. (2000).

In this context, nonlinearities are driven by: (i) endogenous nonlinearities in labour demand from using a multi-input production function and from endogenous vacancy creation; and (ii) exogenous non-linearities in the distribution of workers across ability types.

When calibrated to match the UK labour market, the model is able to replicate, qualitatively and quantitatively, empirical estimates of the profit and wage response to the introduction of the minimum wage in the UK. However, the model predicts a counter-factually large unemployment increase in response to the minimum wage's introduction. This illustrates that the inclusion of search frictions is certainly not a sufficient condition for matching empirical evidence of small/non-existent employment impacts. When considering minimum wage levels above those experienced already, the model suggests a nonlinear unemployment reaction that starts well before the planned level of the minimum wage in 2020.

Quantitatively, I find that imperfect substitution between inputs is the most significant endogenous source of nonlinearities in the model. If we instead use a constant returns to scale production function with labour as the only input, the model predicts that unemployment increases with the minimum wage in a much more linear fashion. This is significant as the search literature on minimum wages generally assumes constant returns to scale production with labour as the only factor of production, and that different worker types are perfectly substitutable (Flinn (2006), van den Berg and Ridder (1998)).

The assumption of constant returns to scale in labour input typically made in the search literature ensures the common restriction that firms employ a maximum of one worker can be made without loss of generality (in the context of the model at least). This assumption of one worker firms avoids the complexity of firm owners bargaining with multiple workers, as described in Stole and Zwiebel (1996). The theoretical contribution of this paper is to develop an internally consistent model of production and the labour market that effectively incorporates both imperfect substitution between labour inputs and wage bargaining, without the complexities of Stole and Zwiebel (1996). I achieve this by confining search frictions to intermediate goods firms, where there is constant returns to scale production using labour inputs only. These intermediate goods firms sell their output to a final good producer that has production technology featuring imperfect substitution between all inputs and capital skill complementarity as per Krusell et al. (2000). This production structure is similar in flavour to Acemoglu (2001), though his focus is on the impact of the minimum wage on the composition of jobs rather than the aggregate unemployment response. The quantitative contribution of this paper is to show that allowing for imperfect substitution between inputs has a significant impact on the nonlinear relationship between the minimum wage and unemployment in my model.

The model I develop captures only the 'disemployment' impacts of the minimum wage, which can only ever have a negative impact on employment rates in the model for workers for whom the minimum wage binds. I do not consider gains in participation from minimum wage increases as discussed in Flinn (2006). There are also other margins of response for firms than the employment margin that I focus on, like changing hours worked or decreasing fringe benefits. In that sense, predictions from the model outlined here could be viewed as a somewhat cautious lower bound estimate of where any unemployment nonlinearities might lie.

The rest of the paper is organised as follows. Section 2 reviews both the search literature featuring minimum wages and the 'reduced form' empirical literature, with a focus on the UK experience. Section 3 sets out the model, and considers the factors determining the employment impacts of the minimum wage and how this differs from more standard competitive and frictional models. Section 4 describes my calibration strategy and assesses whether the calibrated model can match empirical findings concerning the impact of the introduction of the minimum wage in the UK. Section 5 examines the quantitative implications of the model to asses whether there is indeed a nonlinear relationship between unemployment and the minimum wage, and investigates what determines the location and strength of any such nonlinearity. Section 6 concludes.

2. Related Literature

2.1. *Search Literature on Minimum Wages*

The nature of wage setting in frictional models is crucial in determining the impact of minimum wages. Two forms of wage setting are commonly used in the search literature: wage posting, where firms offer a take-it-or-leave it wage, or wage bargaining.

Wage posting models of the type pioneered by Burdett and Mortensen (1998) typically feature pure wage dispersion, i.e. dispersion that is not generated by worker or firm heterogeneity but is the result of a mixed strategy played by rival firms. The presence of pure wage dispersion means minimum wage increases will raise workers' wages with no employment impact as long as the minimum wage remains below the level of match productivity. However, any increases in the minimum wage beyond this point will destroy all such matches due to the common assumption of constant returns to scale in production. Engbom and Moser (2017) find minimum wages have sizable, and realistic, impacts on the wage distribution in a wage posting model that is estimated using data from a large minimum wage increase in Brazil. Although it's not the focus of their paper, their model also predicts a large rise in unemployment in response to the minimum wage increase.

Wage bargaining models with exogenous contact rates, e.g. Flinn (2006), also feature stark unemployment impacts whereby the minimum wage has no impact on employment until it hits the level of match ability, whereupon all matches of this ability level are destroyed. Wage bargaining models that have endogenous vacancy creation - again Flinn (2006) looks at this case - have a more gradual increase in unemployment until the minimum wage reaches the level of match ability at which point again matches are destroyed. The reduction in labour demand before this point occurs because the minimum wage decreases the share of profits going to firms, which disincentivises vacancy creation.

A common assumption running through this literature is that labour has a constant marginal product. This produces the stark "cliff-edge" results discussed above i.e. once the minimum wage exceeds this fixed marginal product, the match is destroyed. A key contribution of the model I present is that it combines search frictions with a production structure exhibiting diminishing marginal product in labour inputs (strictly speaking, the intermediate good produced by labour has a diminishing product in my model, rather than labour itself). This means that even if the current minimum wage exactly equals the marginal product of a match, an increase in the minimum wage need not destroy all such matches as at zero employment labour has an infinite marginal product (i.e. I assume Inada conditions hold).

Haanwinckel (2018) presents a model of the minimum wage featuring imperfect substitution between different worker types, who perform tasks of varying routine skill intensity, and imperfectly competitive labour markets. While his model has rich implications for minimum wage spillovers on the wage distribution, which is the focus of the paper, it captures imperfections in the labour market in a relatively reduced-form way, i.e. through a inelastic labour supply function to firms, and so has less scope to explore unemployment impacts.

2.2. *Empirical Evaluation of Minimum Wage Changes*

This section focuses on studies that evaluate changes to the minimum wage in the UK, as I will calibrate my model to UK data, however I start with a brief discussion of the sizable US evidence base.

A large fraction of US studies focus on the employment response of teenagers to the minimum wage or on particular sectors like fast-food restaurants (Neumark and Wascher (1995), Card and Krueger (1994)). While there is a clear interest in looking at areas where the minimum wage bites hardest, such studies offer little guidance regarding the macroeconomic impacts of the minimum wage, which is the focus of this paper. However, more recent studies such as Cengiz et al. (2018) consider aggregate employment responses to state level minimum wage changes. The employment change induced by an increase in the minimum wage is calculated by comparing the increase in the density of workers paid at or just above the newly increased minimum wage to the decrease in density below the minimum wage. Looking at 138 state level minimum wage changes, they find no evidence of significant employment impacts.

Of course my concern is finding nonlinearities in the impact of the minimum wage on employment. In the U.S context, it is therefore instructive to consider evidence on aggregate employment impacts for states/regions that have introduced particularly high minimum wage levels. Jardim et al. (2017) consider the case of Seattle, where the city authorities raised the local minimum wage from \$9.47 to \$11 in 2015 and to \$13 in 2016. The authors report that they find *"evidence of nonlinear effects, as the rise to \$11 per hour had an insignificant effect on employment, whereas the rise to \$13 per hour resulted in a large drop in employment"*.

One critique of U.S studies is that federal and municipal minimum wage increases tend to be done in nominal terms and are soon eroded by inflation, so that the findings above are more relevant for short term impacts and may not be indicative of long term effects. This is less of a concern in the U.K where, when not in recession, the minimum wage tends to keep track with, or exceed, earnings and prices. Leonard et al. (2014) perform a meta-analysis of studies looking at the employment response to the introduction, and subsequent increases, of the UK minimum wage. They find the mean estimate of the employment elasticity is not significantly different from zero.

There are of course many margins of adjustment available to firms other than employment, e.g. hours worked, non-wage benefits, prices or profits. Taking hours first, there appears to be more evidence of effects through this channel in the UK than with employment, although the estimated reductions in hours have generally not been sufficient to reduce weekly earnings (Stewart and Swaffield (2008), Dickens et al. (2012).and Connolly and Gregory (2002).

Firms facing increases in their labour costs due to the minimum wage may also raise their prices. In their 2014 annual report, the UK Low Pay Commission (henceforth the LPC), who are responsible for recommending the level of the minimum wage to central government, note that prices have risen considerably faster in those sectors where minimum wage workers are concentrated. While this evidence is suggestive of price pass-through it is far from conclusive. Wadsworth (2009) tests this hypothesis in a slightly more formal regression framework and finds evidence of mild, but statistically significant, price pass through. In her survey of the impact of minimum wages on prices, Lemos (2008) comes to similar conclusions. This is consistent with international evidence of price pass through i.e. Harasztosi and Lindner (2015).

The other major avenue for employers to avoid the incidence of increased wages is to reduce non-wage benefits (e.g. pension contributions or bonus payments).

The LPC commissioned research on this which concluded that firms did indeed reduce labour costs by reducing pay premia for overtime and unsocial hours; and by restricting non-wage benefits such as subsidised meals and transport, annual leave, pensions, and staff discounts (Grimshaw and Carroll (2002), Cronin and Thewlis (2004) and Denvir and Loukas (2006)). However, the introduction of default employee enrollment into pension schemes ('auto-enrollment'), with a mandatory contribution from the employer, will limit the extent to which employers can lower pension contributions.

This is of course a piecemeal approach to examining who bears the incidence of minimum wage increases. Arguably a more direct test of this is to examine the impact on firm profitability. This is exactly the approach of Draca et al. (2011) who look at firm profitability for extended periods before and after the introduction of the minimum wage in the UK. They find firms employing relatively large numbers of minimum wage workers have lower profit growth than those employing higher wage workers. The authors also note that the size of profit reduction is consistent with a static model where employers do not change their behaviour in response to the minimum wage change. A final finding is that there is not a statistically significant change in firm exit rates or employment.

In summary, the UK evidence points to muted impacts of previous minimum wage changes on employment which, combined with findings that firms absorbed a substantial amount of the wage increase through reduced profits, is suggestive of monopsonistic labour markets. However, muted short term employment impacts can be reconciled with competitive models of the labour market, as in the putty clay model of Aaronson et al. (2013). Competitive models are also consistent with findings of price pass-through and reductions in employee benefits and hours. This guides my choice of modeling assumptions in that, while I allow for some monopsony power by assuming search frictions and wage bargaining, I also allow for employer competition by assuming workers can search OTJ and that employers can respond to poaching by rivals as in Cahuc et al. (2006).

3. The Model

3.1. *Model Environment*

Model Environment: Workers

There are two skill types of workers, unskilled and skilled, and within each skill type there is a distribution of worker ability, with skill indexed by h and ability indexed by i. A worker of skill type h and ability type i has an efficiency level, which we will define precisely later, denoted by $x_{h,i}$. We assume a worker's skill type is observable to the researcher and firms, but their ability is observable to firms only. Workers' ability levels are distributed within a skill type according to the cdf $L_h(x)$, and pdf $\ell_h(x)$. For quantitative purposes later I will use a discrete approximation to a log normal ability distribution, with a total of M ability types for each skill group (I set $M = 400$ in my baseline calibration). All workers and firm owners have a common discount factor, β, and are risk neutral.

For notational convenience, the subscript j - which I refer to as a worker type - will be a vector valued index containing both the skill index ($h \in \{u, s\}$) and ability index ($i \in \{1..M\}$) of a worker.

Model Environment: Production Structure

The production structure in my model has two layers. First there is an intermediate goods sector with search frictions, where I maintain the typical assumptions of the search literature (no capital and constant returns to scale production in labour inputs). Second, I include a final good sector with a production function that combines intermediate goods with capital, and features imperfect substitutability of all factors and capital skill complementarity as per Krusell et al. (2000) (henceforth referred to as the "KORV" production function).

There will be a segmented intermediate goods sector for each of the $2M$ possible pairings of skill type ($h \in \{u, s\}$) and ability level ($i \in \{1..M\}$). Firms in these intermediate sectors can be thought of as hiring agencies for the final goods firm, that face search frictions and wage bargaining. This economy is represented in stylised form in Figure 2.

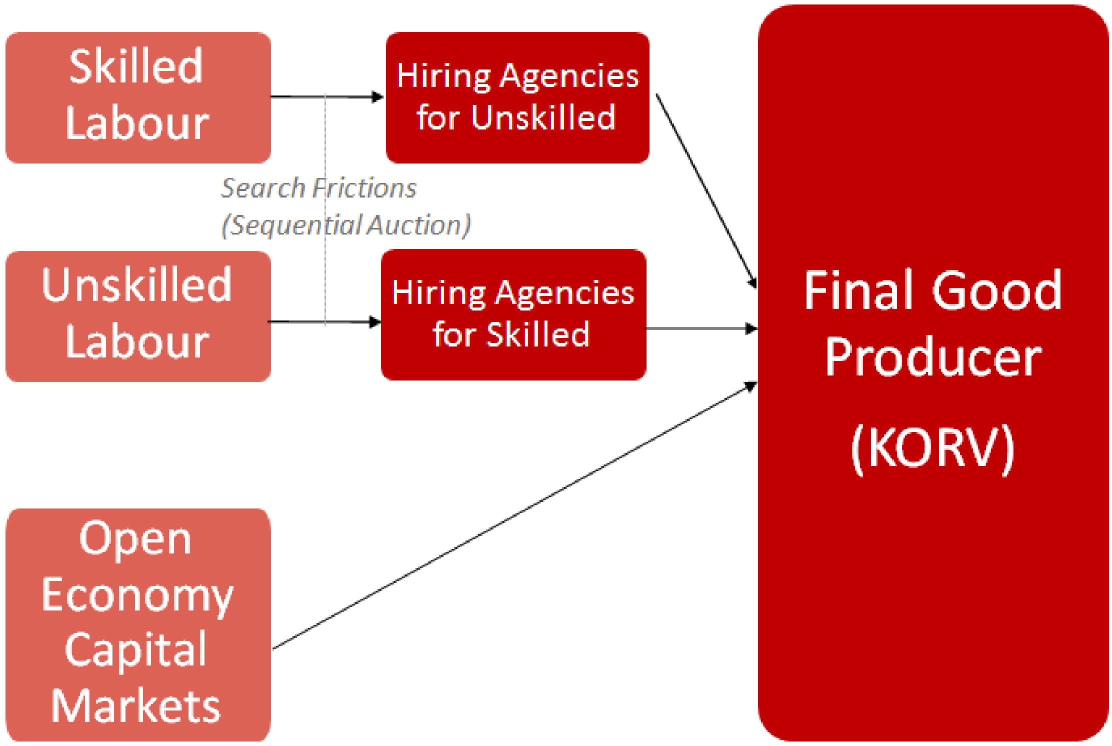
I do not introduce search frictions and wage bargaining directly into the final good production stage as that would involve firms bargaining with many workers, e.g. a multi-player game, as described in Stole and Zwiebel (1996). In this environment, each worker would consider the impact of their negotiation on the negotiations of all other workers. Such considerations do not feel particularly relevant for investigating the macroeconomic impacts of minimum wages, so I choose to abstract from these effects using the production structure described above.

Model Environment: Final Good Firms

I use the same production structure as in Krusell et al. (2000), which is shown in equation (1). Final goods are produced using capital structures, K_{st}, capital equipment, K_{eq}, and aggregates of the intermediate goods produced by unskilled and skilled workers; these aggregate inputs are denoted U and S respectively.[1] U and S are aggregates of the output of the M types of intermediate goods firm

[1]Krusell et al. (2000) assume a perfectly competitive labour market, and U and S are simply the total hours worked by each skill group.

FIGURE 2. Model Economy Overview



in each skill sector, which correspond to the M heterogeneous ability levels of unskilled and skill workers. The output of the intermediate good sector employing unskilled (skilled) labour of ability type i is denoted $y_{u,i}$ $(y_{s,i})$.

$$\begin{aligned} Y &= AG(K_{st}, K_{eq}, U, S) \\ &= AK_{st}^{\alpha}[\mu U^{\sigma} + (1-\mu)(\lambda K_{eq}^{\rho} + (1-\lambda)S^{\rho})^{\frac{\sigma}{\rho}}]^{\frac{1-\alpha}{\sigma}} \end{aligned} \tag{1}$$

$$U = \left(\sum_{i=1}^{M}(x_{u,i}y_{u,i})^{\frac{\Psi_u-1}{\Psi_u}}\right)^{\frac{\Psi_u}{\Psi_u-1}}, \; S = \left(\sum_{i=1}^{M}(x_{s,i}y_{s,i})^{\frac{\Psi_s-1}{\Psi_s}}\right)^{\frac{\Psi_s}{\Psi_s-1}} \tag{2}$$

with $\sigma, \rho < 1$, $\alpha, \lambda, \mu \in (0,1)$ and $\Psi_u, \Psi_s > 1$. The elasticity of substitution between the aggregate unskilled intermediate input and capital equipment, denoted by $\varepsilon_{u,k_{eq}}$, equals $1/(1-\sigma)$. The elasticity of substitution between the unskilled intermediate input and the skilled intermediate input, denoted $\varepsilon_{u,s}$, is also given by $1/1-\sigma$. The elasticity of substitution between the skilled intermediate input and capital equipment, denoted by $\varepsilon_{s,k_{eq}}$, is given by $1/(1-\rho)$. The parameter α, together with λ, determine the capital share of output, and μ impacts the output share of unskilled intermediate good sectors. The production function will exhibit capital skill complementarity, i.e $\varepsilon_{u,k_{eq}} > \varepsilon_{s,k_{eq}}$, whenever $\sigma > \rho$. This is exactly what Krusell et al. (2000) find to be the case and I will use their parameter estimates (I discuss this further in Section **??**).

Equation (2) states that the efficiency level of a worker of skill type h and ability type i, $x_{h,i}$, corresponds to the efficiency of the intermediate good they produce in final good production.

Model Environment: Intermediate Goods Sectors

There is a separate intermediate goods sector for each worker type j (recall j is a vector index of skill and ability: $j = (h,i)$) and one intermediate firm for each worker in the economy, so the density of intermediate goods firms in sector j equals the population density of workers ℓ_j. I assume all intermediate firms sell competitively to the final good firm. Intermediate goods sectors are completely segmented in the sense that a type j firm can only ever employ a type j workers and vice versa.

The assumption of constant returns to scale in intermediate good sectors means the output of sector j (y_j) will simply be the population density of type j workers multiplied by their employment rate and hours worked, $\bar{H}$ i.e. ($y_j = \ell_j(1-e_j^{ue})\bar{H}$), where e_j^{ue} denotes the unemployment rate of type j workers. I include hours worked as the KORV production function was originally specified with labour input measured in terms of total hours, however, I assume both worker types are full-time, i.e. work a fixed 40 hour week, and do not model the intensive margin of labour supply.

Model Environment: Search Frictions and Wage Bargaining in the Intermediate Goods Sectors

I assume that both unemployed and employed workers randomly search for jobs. The homogeneity of intermediate goods firms means workers exist in one of three employment states: unemployed; employed but not yet poached by another employer ('not-poached'); or employed and poached ('poached'). The employment state for a worker of skill type j is denoted as $\Upsilon_j \in \{ue, np, p\}$, where the indices $\{ue, np, p\}$ represent the unemployed, not-poached and poached employment states respectively.

The number of newly formed job matches is given by matching function $M(S_j, V_j)$, where S_j is the effective number of type j job searchers (unemployed and not-poached workers) and V_j is the number of type j vacancies. I assume that unemployed workers search more intensely than non-poached workers so that $S_j = N_j^{ue} + \chi_j N_j^{np}$, where N_j^{ue} is the number of unemployed type j workers,

N_j^{np} is the number of not-poached workers, and χ_j is the search intensity rate for employees relative to the unemployed ($\chi > 0$). Once a worker is poached they stop searching as all firms are the same.

Defining $\theta_j \equiv V_j/S_j$ as labour market tightness, the contact rate is $q(\theta_j) \equiv M(S_j, V_j)/V_j$ for type j firms, and $(\theta_j q(\theta_j), \chi_j \theta_j q(\theta_j))$ for type j unemployed and not-poached workers respectively. The fraction of type j workers who are poached is denoted by e_j^p and the fraction who are not-poached by e_j^{np} (with the residual fraction unemployed denoted by e_j^{ue}). The share of effective job searching workers that are not-poached is denoted as $s_j^{np} \equiv \frac{\chi_j e_j^{np}}{\chi_j e_j^{np} + e_j^{ue}}$, and the share that are unemployed as $s_j^{ue} \equiv 1 - s_j^{np}$. Finally matches are destroyed with exogenous probability, δ_j.

I follow the approach of Cahuc et al. (2006) where all firms and workers engage in Nash bargaining. For unemployed workers matched with a firm, who then become 'not-poached' workers in my terminology, standard Nash bargaining takes place. This bargaining is subject to the constraint that the bargained wage must be at least as large as the legally binding minimum wage, m_w.

When a not-poached worker makes contact with another employer, becoming a poached worker, they also engage in Nash bargaining but this time the bargain is between the incumbent and poaching employer and the worker, as in Cahuc et al. (2006). The rival employers bid-up the wage until the value of employing a poached worker to the firm equals the value of carrying a vacancy. Free entry will drive the latter to zero, due to the existence of a fixed vacancy cost κ_j. As type j firms are a priori identical, the poaching firm will offer the same wage as the incumbent (which we will see is the price of the intermediate good) leaving the worker indifferent between the two rival firms.

I arbitrarily assume the worker moves with probability one to a poaching firm conditional on making contact with them. This assumption means job contact rates, which are unobservable in the data, are equal to job mobility rates, which are observable. If in reality job contact rates for employees were significantly greater than job mobility rates, then this effectively moves the model closer to a competitive labour market or equivalently one with higher worker bargaining power. In that sense bargaining power and the probability of moving to a poaching firm conditional on contact with them are not separately identifiable. I therefore estimate the former and set the latter equal to one as a normalisation.

3.2. *Behaviour in the Model Economy*

Behaviour: workers

A worker of a given type j exists in one of three employment states: unemployed and receiving flow income b, employed but not poached and receiving the higher of the Nash bargained wage w_j^b and the minimum wage m_w, or employed and poached and receiving wage w_j^p. The expected lifetime utility of being in each of these states will be denoted by V_j^{ue}, V_j^{np} , and V_j^p respectively.

Workers face only one trivial decision: whether to participate in the labour market which they do as long as they are paid more than their reservation wage. Given vacancies are costly, rational firms will always offer at least the reservation wage so this decision is trivial. The Bellman equations for a unemployed, not poached and poached worker are therefore as follows:

$$V_j^{ue} = b + \beta[\theta_j q(\theta_j)V_j^{np} + (1 - \theta_j q(\theta_j))V_j^{ue}] \tag{3}$$

$$V_j^{np} = \max(w_j^b, m_w) + \beta\big[\delta_j V_j^{ue} + (1 - \delta_j)\big[\chi_j \theta_j q(\theta_j)V_j^p + (1 - \chi_j \theta_j q(\theta_j))V_j^{np}\big]\big] \tag{4}$$

$$V_j^p = w_j^p + \beta[\delta_j V_j^{ue} + (1 - \delta_j)V_j^p] \tag{5}$$

Equation (3) states that an unemployed worker of skill level j receives benefits, b, in the current period and in the next period either gets a job offer with probability $\theta_j q(\theta_j)$, which they will always accept and so become a not poached worker, or remains unemployed with probability $1 - \theta_j q(\theta_j)$. Equation (4) states that a not poached worker gets the higher of the Nash bargained wage and the minimum wage in the current period and in the following period loses their job with probability δ_j , gets poached with probability $(1 - \delta_j)\chi\theta_j q(\theta_j)$ or remains not poached with probability $(1-\delta_j)(1-\chi\theta_j q(\theta_j))$. Finally equation (5) states that a poached worker gets a wage w_j^p in the current period and the next period either loses their job with probability δ_j or remains employed as a poached worker (since they have already reached the top of the job ladder) with probability $1 - \delta_j$.[2]

[2]We will see later that poached workers are paid a wage equal to the price of the intermediate good they produce. This is equal to the marginal product of the intermediate good, which will always exceed the minimum wage, $w_j^p = p^j >= m_w$: if this were not the case intermediate firms would be loss making and leave the market, until the price of the intermediate good is bid up by the final good producer to the level of the minimum wage (Inada conditions guarantee this point will be reached)

Behaviour: Final Good Producers

The final good producer's profit maximisation problem is as follows, where we normalise the price of the final good to one:

$$\begin{aligned}
&\max_{K_{st},K_{eq},y_{1,u}\cdots y_{M,u},y_{1,s}\cdots y_{M,s}} \Pi = AK_{st}^{\alpha}[\mu U^{\sigma} + (1-\mu)(\lambda K_{eq}^{\rho} + (1-\lambda)S^{\rho})^{\frac{\sigma}{\rho}}]^{\frac{1-\alpha}{\sigma}} \\
(6) \qquad &- \sum_{i=1}^{M} p_{i,u}y_{u,i} - \sum_{i=1}^{M} p_{i,s}y_{s,i} - r_{st}K_{st} - r_{eq}K_{eq} \\
&U = \left(\sum_{i=1}^{M}(x_{u,i}y_{u,i})^{\frac{\Psi_u-1}{\Psi_u}}\right)^{\frac{\Psi_u}{\Psi_u-1}}, S = \left(\sum_{i=1}^{M}(x_{s,i}y_{s,i})^{\frac{\Psi_s-1}{\Psi_s}}\right)^{\frac{\Psi_s}{\Psi_s-1}}
\end{aligned}$$

As in Krusell et al. (2000), I impose a no arbitrage condition between capital equipment and capital structures. This implies that the net of depreciation rental rates for capital equipment and structures must be equal to some common interest rate, r, which implies their gross rental rates, r_{eq} and r_{st}, are related as follows: $r_{eq} - \delta_{eq} = r_{st} - \delta_{st} = r$, where δ_{eq} and δ_{st} are the depreciation rates for capital equipment and structures respectively.[3] I assume the final goods sector is competitive, and that intermediate goods sectors sell their output competitively, meaning factors of production are paid their marginal products as shown in equations (7) through to (10).

$$\begin{aligned}
p_{i,u} &= A(1-\alpha)K_{st}^{\alpha}[\mu U^{\sigma} + (1-\mu)(\lambda K_{eq}^{\rho} + (1-\lambda)S^{\rho})^{\frac{\sigma}{\rho}}]^{\frac{1-\alpha-\sigma}{\sigma}} && (7) \\
&\times \mu U^{\sigma-1}\left(\sum_{i=1}^{M}(x_{u,i}y_{u,i})^{\frac{\Psi_u-1}{\Psi_u}}\right)^{\frac{1}{\Psi_u-1}}(x_{u,i}y_{u,i})^{\frac{-1}{\Psi_u}}x_{u,i} \\
p_{i,s} &= A(1-\alpha)K_{st}^{\alpha}[\mu U^{\sigma} + (1-\mu)(\lambda K_{eq}^{\rho} + (1-\lambda)S^{\rho})^{\frac{\sigma}{\rho}}]^{\frac{1-\alpha-\sigma}{\sigma}} \times && (8) \\
&(1-\mu)(\lambda K_{eq}^{\rho} + (1-\lambda)S^{\rho})^{\frac{\sigma-\rho}{\rho}}(1-\lambda)S^{\rho-1} \times \\
&\left(\sum_{i=1}^{M}(x_{s,i}y_{s,i})^{\frac{\Psi_s-1}{\Psi_s}}\right)^{\frac{1}{\Psi_s-1}}(x_{s,i}y_{s,i})^{\frac{-1}{\Psi_s}}x_{s,i} \\
r_{eq} &= A(1-\alpha)K_{st}^{\alpha}[\mu U^{\sigma} + (1-\mu)(\lambda K_{eq}^{\rho} + (1-\lambda)S^{\rho})^{\frac{\sigma}{\rho}}]^{\frac{1-\alpha-\sigma}{\sigma}} \times && (9) \\
&(1-\mu)(\lambda K_{eq}^{\rho} + (1-\lambda)S^{\rho})^{\frac{\sigma-\rho}{\rho}}K_{eq}^{\rho-1} \\
r_{st} &= \alpha AK_{st}^{\alpha-1}[\mu U^{\sigma} + (1-\mu)(\lambda K_{eq}^{\rho} + (1-\lambda)S^{\rho})^{\frac{\sigma}{\rho}}]^{\frac{1-\alpha}{\sigma}} && (10)
\end{aligned}$$

[3] When it comes to calibrating the model I will assume that both net of depreciation rates equal the natural rate of interest $r = \frac{1}{\beta} - 1$.

Behaviour: Intermediate Goods Producers

Intermediate firms are either inactive, generating zero expected liftetime utility for their owners (I refer to the expected lifetime utility of firm ownership as the firm's value), or exist in one of three active states: (i) carrying a vacancy, with a firm value denoted by J_j^v; (ii) employing a not-poached worker, with a firm value denoted by J_j^{np}; or (iii) employing a poached worker at a wage w_j^p, with a firm value denoted by J_j^p. The corresponding bellman equations are as follows:

$$J_j^v = -\kappa_j + \beta[q(\theta_j)\{s_j^{ue} J_j^{np} + (1 - s_j^{ue}) J_j^p\} + (1 - q(\theta_j)) J_j^v] \tag{11}$$

$$J_j^{np} = p_j - \max(w_j^b, m_w) + \beta\left[(1 - \delta_j)\{\chi_j \theta_j q(\theta_j) J_j^p + (1 - \chi_j \theta_j q(\theta_j)) J_j^{np}\} + \delta_j J_j^v\right] \tag{12}$$

$$J_j^p = p_j - w_j^p + \beta[(1 - \delta_j) J_j^p + \delta_j J_j^v] \tag{13}$$

Equation (11) states that a firm in intermediate good sector j carrying a vacancy pays a vacancy cost, κ_j, in the current period and in the next period makes contact with an unemployed worker with probability $q(\theta_j)s_j^{ue}$, makes contact with an employed worker with probability $q(\theta_j)(1 - s^{ue})$, or remains carrying a vacancy with probability $1 - q(\theta_j)$. Equation (12) states that a firm employing a not poached worker gets profits $p_j - \max(w_j^b, m_w)$ in the current period and in the next period remains employing that worker with the probability $(1 - \delta_j)(1 - \chi\theta_j q(\theta_j))$, loses the worker to a rival firm with probability $(1 - \delta_j)\chi\theta_j q(\theta_j)$, or the job is destroyed with probability δ_j. Finally equation (13) states that a firm employing a poached worker gets profit $p_j - w_j^p$ in the current period and in the next period the job is either destroyed with probability δ_j or they remain employing the poached worker with probability $1 - \delta_j$.

Free entry into markets by inactive firms will drive the value of holding a vacant job, J_j^v, to zero, and competition between employers drives the value of employing a poached worker to the value of holding vacancy e.g. $J_j^p = 0$ too. The free entry condition ($J_j^v = 0$) and poaching condition ($J_j^p = 0$) imply the poached wage, w_j^p equals the price of the intermediate good p_j.

Equations (11) and (12), combined with the free entry condition, imply:

$$\kappa_j = \beta q(\theta_j) s_j^{ue} \frac{p_j - \max(w_j^b, m_w))}{1 - \beta(1 - \delta_j)(1 - \chi_j \theta_j q(\theta_j))} \tag{14}$$

Inactive intermediate firms enter the market, by posting a new vacancy, until the discounted expected profits from hiring a worker equal the vacancy cost. This discounting reflects both the discount factor and the risk that the worker will be exogenously seperated from the firm (with probability δ_j) or be poached by another firm (with probability $\chi_j \theta_j q(\theta_j)$).

The Nash bargained wage is determined in the standard maximisation problem, shown in equation (15).

$$
\begin{aligned}
w_j^b &= \underset{w_j^b}{\operatorname{argmax}} (V_j^{np} - V_j^u)^{\phi_j} (J_j^{np})^{1-\phi_j} \\
&= \Phi_j p_j + (1 - \Phi_j)\big(V_j^u(1-\beta) - \beta(1-\delta_j)\chi_j\theta_j q(\theta_j)(V_j^p - V_j^u)\big) \qquad (15)
\end{aligned}
$$

The fact that the minimum wage acts as a side constraint on the Nash bargained wage implies that, in the absence of equilibrium impacts on prices of intermediate goods or contact rates, there are no "spillover" impacts of the minimum wage. Only workers with initial wages lower than the minimum wage benefit from its imposition, and will see their wages bid up to the value of the minimum wage and no higher.[4][5] However, once I allow for equilibrium effects, such as changes to the prices of intermediate goods or contact rates, the absence of minimum wage spillovers is no longer a given. While this paper focuses on the unemployment impact of minimum wages, Appendix B discusses their impact on the shape of the wage distribution.

3.3. *Equilibrium*

One condition for a steady state equilibrium in the model, which I will formally define later, is that the labour market is in steady state. This requires the following equations to hold:

$$\delta_j(1 - e_j^{ue}) = \theta_j q(\theta_j) e_j^{ue} \qquad (16)$$

$$\theta_j q(\theta_j) e_j^{ue} = (\delta_j + (1-\delta_j)\chi_j\theta_j q(\theta_j)) e_j^{np} \qquad (17)$$

Equation (16) equates inflows into unemployment (LHS of the equation) to outflows (RHS), where the inflow consists of employees losing their jobs, with probability δ_j, and the outflow is unemployed workers gaining jobs, with probability $\theta_j q(\theta_j)$. Similarly equation (17) equates the inflow in of workers into the not-poached state (LHS) with the outflow (RHS), where the inflow consists of unemployed workers gaining employment with probability $\theta_j q(\theta_j)$, and the outflow (RHS) is not-poached workers either losing their job, with probability δ_j, or becoming poached, with probability $(1-\delta_j)\chi_j\theta_j q(\theta_j)$.

[4]I need that the solution to the Nash maximisation both exists and is the unique global maximum to justify my assertion that the wage outcome as the higher of the Nash wage and minimum wage. However, this is given from the linearity of all value functions and compactness of the feasible set.

[5]This would not necessarily be the case if I had used alternative bargaining solutions such as the Kalai-Smoridinsky solution concept (see e.g. Dittrich and Knabe (2013)). It is also will generally not be true when there is match heterogeneity, and the minimum wage may influence the reservation match quality accepted by workers (see Flinn (2003)).

The steady-state Equations (16) and (17) combined with a specification of the matching function can provide an expression for the steady-state level of labour market tightness as a function of the unemployment rate, which I denote as $\theta^{ss}(e_j^{ue})$ respectively. I derive an inverse supply function for intermediate goods, shown in equation (18), from these steady state conditions and the no entry condition in the intermediate good sector. The demand price equation comes from the first order conditions of the final good producer's first order conditions, as shown in equation (19).

$$(18) \qquad p_j^s = \max(w_j^b, m_w) + \frac{\kappa_j \Big(1 - (\beta(1-\delta_j)(1-\chi_j\theta^{ss}(e_j^{ue})q(\theta^{ss}(e_j^{ue}))))\Big)}{\beta q(\theta^{ss}(e_j^{ue}))s_j^{ue}}$$

$$(19) \qquad p_j^d = \frac{\partial Y}{\partial y_j(e_j^{ue})}$$

The supply price of intermediate goods is the sum of the wage payment to a type j not-poached worker and discounted expected vacancy costs. The demand price is simply the marginal product of the intermediate good. The prices of intermediate goods follow from equating supply (from intermediate good producers) and demand (from the final good producer).

Definition 1. *The recursive stationary equilibrium consists of,* $\forall j \in \{(u,1)..(u,M),(s,1)..(s,M)\}$ *and for a fixed interest rate,* r*, and minimum wage,* m_w*:*

(i) a set of worker value functions $\{V_j^{ue},\, V_j^{np},\, V_j^{p}\}$*,*

(ii) a set of firm value functions $\{J_j^v, J_j^p, J_j^{np}\}$*,and vacancies,* v_j*,*

(iii) a set of employment states $\{e_j^{ue}, e_j^{np}, e_j^{p}\}$*,*

(iv) a choice of capital equipment, capital structures, and intermediate goods (K_{eq}, K_{st}, y_j) *by the final good producer*

(v) prices $\{p_j,\, w_j^b,\, w_j^p\}$ *; which satisfy:*

(1) Worker Optimisation:
The worker value functions satisfy equations (3)*,* (4) *and* (5)*.*

(2) Final Good Producer Optimisation:
The final good producer's choice of capital equipment and structures, K_{eq} *and* K_{st} *and intermediate goods* y_j *satisfy the FOCs* (7) *through to* (10) *.*

(3) Steady State in the Intermediate Good Sector:
The no-entry condition, 14, and steady state conditions 16 and 17 are met.

(4) *Intermediate Goods Market Clearing:*
Demand and supply for each intermediate good must be equal, implying conditions 18 and 19 hold simultaneously.

(5) *Wage Determination:*
Not poached workers are paid the higher of the Nash bargained wage wage w_j^b*, as specified in equation* (15)*, and the minimum wage,* m_w*. Poached workers are paid the competitive wage,* $w_j^p = p_j$

(6) *Consistency:*
Given employment and vacancy rates, the job contact rates determined by the matching function are consistent with those used in the worker and firm optimisation problems.

3.4. *Minimum Wage impacts on Unemployment*

Equations (18) and (19), imply that the equilibrium wage paid to a not-poached worker equals the marginal productivity of the intermediate good they produce minus expected recruitment costs at the equilibrium unemployment rate, e_j^{ue*}, and corresponding level of labour market tightness, $\theta(e_j^{ue*})$:

$$(20) \quad \max(w_j^b, m_w) = \frac{\partial Y}{\partial y_j(e_j^{ue*})} - \frac{\kappa_j\Big(1-(\beta(1-\delta_j)(1-\chi_j\theta(e_j^{ue*})q(\theta(e_j^{ue*}))))\Big)}{\beta q(\theta(e_j^{ue*}))s_j^{ue}}$$

The employment impacts of a minimum wage increase are unambiguously negative due to two features of the model. First, the marginal product of an intermediate good, and hence its price, are decreasing in the amount of intermediate good used. Second, recruitment costs are increasing in the steady state employment rate. This holds because extra vacancy creation is needed to sustain a higher employment rate, which results in a reduced vacancy filling rate $q(\theta_j)$ and higher recruitment costs. However, an important implication of equation (20) is that the employment impact of the minimum wage in my model will be more muted than in a comparable model with perfect competition and no labour market frictions, and compared to a model with with a more typical frictional labour market structure (i.e. CRS production with labour as the only factor input).

In a comparable competitive model, an increase in the marginal product via reduced employment levels is the only force that can restore equilibrium in the labour market following a minimum wage increase. Adding frictional labour markets to this set-up means the fall in employment necessary to restore equilibrium is less as recruitment costs fall when employment is reduced.

The same logic applies when I compare my model to a more typical frictional benchmark. When labour effectively has a constant marginal product, the fall in recruitment costs is the only force that can restore equilibrium following a minimum wage increase. Adding a production function where labour produces an intermediate good with a decreasing marginal product means the fall in employment necessary to restore equilibrium is again less as the marginal product of the intermediate good now increases when employment falls.

3.5. *Solution Algorithm*

For a fixed world interest rate, r, I:

(1) Guess the unemployment rate e^{ue}_{j0}, $\forall j \in \{(u,1)..(u,M),(s,1)...(s,M)\}$.

(2) Use this guess to construct the aggregate output of intermediate goods produced in the unskilled and skilled intermediate sectors (these aggregate outputs, U and S, are defined in equation (2)).

(3) Solve the final good firm's FOCs (equations (9) and (10)) to get their optimal choice of capital equipment and structures, K_{eq} and K_{st}, that is consistent with the implied levels of U and S from above and firm optimisation given the interest rate r. Then derive the price of each intermediate good p_j that is consistent with firm optimisation at the unemployment guess e^{ue}_{j0} using the FOCs in equations (7) and (8).

(4) Use the conditions (16) and (17) to derive vacancy levels necessary for the unemployment guess $e^{ue}_{j,0}$ to be consistent with steady state in the labour market. This then implies employment transition probabilities for the unemployed and employed via the matching function: $\theta_j q(\theta_j)$ and $\chi_j \theta_j q(\theta_j)$ respectively.

(5) Use employment transition probabilities from above and condition that poached worker is paid $w^p_j = \max(p_j, m_w)$ to solve worker value functions and Nash bargained wage using equations (3) to (5) and (15) respectively. Wage of not-poached worker is whatever is highest of this bargained wage and minimum wage. [6]

(6) Use wage levels from above step to give an updated unemployment guess, e^{ue}_{j1}, $\forall j \in \{(u,1)..(u,M),(s,1)...(s,M)\}$ that simultaneously solves free entry condition (14) for the intermediate firm and the final good firm's FOC i.e. equations (18) and (19).

(7) Repeat iteration until convergence of unemployment guess.

[6] As argued previously, we will always have $p_j (= w^p_j) \geqslant m_w$ in equilibrium however this does not necessarily hold outside of equilibrium so I must impose that $w^p_j == \max(p_j, m_w)$ when solving the model.

4. Estimation

This section first describes my estimation strategy, before presenting parameter estimates and examining the model's fit to targeted and non-targeted empirical moments. In particular, I will consider two types of non-targeted empirical moments:(i) macro moments i.e. labour and profit share of output, the capital-output ratio, average firm mark-ups and fit of the wage distribution; and (ii) micro moments i.e. reduced form evidence from the introduction of the minimum wage in the UK (principally Draca et al. (2011)).

As the model moments are largely intractable, and therefore simulated numerically, I do not provide formal identification arguments but instead examine the relationship between the parameters I estimate and the model moments used in their estimation: this is done in Appendix A. I also discuss the logic of choosing the empirical targets I use in the section below.

4.1. *Estimation Strategy*

I will take all but one of the parameters of the final good production function from Krusell et al. (2000). This means applying parameter estimates from a model with a competitive labour market to my model that assume labour market frictions. However, results from a companion paper suggest the parameter estimates obtained by Krusell et al. (2000) are robust to allowing for labour market frictions. This provides some reassurance that applying their parameter estimates to the model developed here is not unreasonable. There is a seperate issue that the estimates that Krusell et al. (2000) provide are based on calibration to the US economy, and I will be calibrating my model to the UK. However, this again seems reasonable as a calibration approach given the UK has exhibited similar, if not identical, trends in wage inequality and in the labour share to the U.S, particularly in the 1980s and 1990s.

I use the matching function specification, and the associated parameter estimate, from Hagedorn and Manovskii (2008b) - $M(u, v) = uv/(u^\gamma + v^\gamma)^{1/\gamma}$, which ensures job contact rates are bounded between zero and one.

I focus on estimating: (i) the parameters in the exogenous distributions of worker ability, with separate distributions for unskilled and skilled labour (which I interpret as non-graduate and graduates respectively); (ii) the elasticities of substitution between workers within these two skill types, ψ_u, ψ_s; (iii) recruitment costs, κ_u, κ_s, which I assume are fixed within each skill type of worker; (iv) bargaining parameters, Φ_u, Φ_s, which I also assume are fixed within each skill type of worker; and (v) the share parameter, μ, in the KORV production function.

I assume a log normal distribution for worker ability within each skill type, meaning in principle there are two distributional parameters to estimate for each skill type i.e the mean and variance parameters, ζ_h and η_h respectively for $h \in \{u, s\}$. I normalise the mean of the ability distribution to one for skilled and unskilled workers but allow differing scale parameters η_u, η_s. This normalisation is justified on the basis that I will instead estimate the share parameter, μ, in the final good production function and TFP.[7] I denote the parameters to be estimated as $\Phi = (\psi_u, \psi_s, \kappa_u, \kappa_s, A, \phi_u, \phi_s, \eta_u, \eta_s, \mu)$.

The remaining parameters, denoted by Ω, are taken from the literature, directly from the data or are set at their legislative levels, as detailed in Table 1. I calibrate the model to data from 2013-14, as this precedes the significant increases in the minimum wage that started in 2014-15 and are planned to end when the minimum wage reaches 60% of the median wage in 2020-21. I assume job destruction rates are fixed within a given skill type but vary between skill types, whereas I assume unemployment income is paid at a fixed rate that is common for all workers.[8]

I estimate the parameters in Φ by simulated method of moments, targeting the following empirical moments for non-graduates and graduates: median wages, variance of log wages, p90/10 and p50/10 ratios and the proportion of unskilled and skilled workers being paid at or less than the minimum wage (I refer to this moment as the minimum wages coverage).[9] The absolute magnitudes of median wages help to discipline the TFP parameter, A, and their relative magnitudes will discipline the output share parameter, μ. Unemployment rates for the unskilled and skilled are informative of both vacancy costs (κ_u, κ_s) and the elasticities of substitution between workers of heterogeneous ability within each skill group (ψ_u, ψ_s). This follows because the vacancy costs influence the unemployment rate of all workers within a given skill type, and the elasticity of substitution influences the unemployment impact of a given minimum wage on low ability workers within a given skill type. I use several measures of wage dispersion i.e. log wage variance, p90/10 and

[7] μ plays an important role in determine the skill premium in the model (see Appendix A) and hence is not separately identifiable from the relative level of the mean ability parameters, ζ_s/ζ_u. Similarly TFP determines average wages in the model, and so is not separately identifiable from the absolute values of the parameters ζ_s, ζ_u. I therefore normalise the mean parameters of the ability distributions in absolute and relative terms.

[8] Unlike in many other jurisdictions, the main form of unemployment benefits in the UK is paid at a flat rate, as under my baseline calibration, rather than as a fixed percentage of previous earnings. Of course, workers may have access to other forms of insurance: in a related paper I consider minimum wage impacts when workers can self-insure themselves through asset accumulation.

[9] I allow for measurement error in minimum wage coverage in two ways. First I count anyone earning less than the minimum wage in my coverage statistic, and include anyone earning within 20 pence over the minimum wage in the data and model as being covered.

Table 1. Calibrated Parameters

Parameter	Description	Source	Value
δ_u	Job destruction rate: unskilled	LFS 2013q4-2014q3	0.011
δ_s	Job destruction rate: skilled	LFS 2013q4-2014q3	0.007
χ_u	Relative search intensity of employed to unemployed: unskilled	LFS 2013q4-2014q3 (ratio of employer change rate to unemployment exit)	0.112
χ_s	Relative search intensity of employed to unemployed: unskilled	LFS 2013q4-2014q3 (ratio of employer change rate to unemployment exit)	0.075
b	Monthly Unemployment benefits (job seekers allowance)	Legislative level 2013-14	313.492
m_w	Hourly minimum wage	Legislative level 2013-14	6.31
σ	Elasticity of substitution between unskilled and skilled workers	Krusell et al. (2000)	0.401
ρ	Elasticity of substitution between skilled workers and capital equipment	Krusell et al. (2000)	-0.495
α	Capital Structures Parameter	Krusell et al. (2000)	0.117
λ	Input share parameter for capital equipment and skilled labour	Krusell et al. (2000)	0.3
γ	Matching Parameter	Hagedorn and Manovskii (2008a)	0.407
β	Monthly discount factor for workers and firms	By assumption	0.996

p50/10 wage ratios, to help pin down the variance parameters η_u, η_s and because they are also informative of vacancy costs (which determine the proportion of not-poached and poached workers). Finally, I use the minimum wage coverage rates for the unskilled and skilled as they help to discipline the bargaining parameters (ϕ_u, ϕ_s).

Equation (21) summarises the estimation method, where $\hat{M}$ denotes a vector of the empirical moments given above, $M(\Phi, \Omega)$ denotes the model predictions of these moments for given choice of estimated and calibrated parameters, and W is the weighting matrix.[10]

$$\Phi^* = \underset{\Phi}{\operatorname{argmin}}(M(\Phi,\Omega) - \hat{M})'W(M(\Phi,\Omega) - \hat{M}) \quad (21)$$

[10]The weighting matrix W, is chosen so I effectively minimise the percentage deviation of model moments from their empirical moments, which avoids the scale of absolute moment deviations biasing estimates i.e. $W = I.\frac{1}{\hat{M}}$.

Table 2. Estimation Results

Moment	**Model Moment**	**Empirical Moment**	**% Deviation (Model - Data)**
Median Hourly Wage: Unskilled	9.85	9.5	3.61
Median Hourly Wage: Skilled	16.08	15.71	2.41
Var Log Wages: Unskilled	0.45	0.49	-7.97
Var Log Wages: Skilled	0.54	0.57	-5.33
p90/50 Wages: Unskilled	2.02	1.92	4.99
p90/50 Wages: Skilled	2.03	1.96	3.35
p50/10 Wages: Unskilled	1.56	1.57	-0.54
p50/10 Wages: Skilled	2.08	2.07	0.7
Min Wage Coverage: Unskilled	0.16	0.16	-0.77
Min Wage Coverage: Skilled	0.06	0.06	-0.56
Unemployment: Unskilled	0.07	0.07	0.95
Unemployment: Skilled	0.03	0.03	0.92

4.2. *Estimation Results*

Table 2 summarises the ability of the model to match its empirical targets. Given I have over identification (10 parameters vs 12 moments), the fact that the maximum absolute deviation is just above 8% is reassuring. The estimated parameters are shown in Table 3. It is perhaps counter-intuitive that my estimation delivers lower elasticities of substitution and higher bargaining parameters for unskilled workers compared to skilled workers. Both results are explained by the fact that the minimum wage bites further into the wage distribution of unskilled workers than skilled workers; in the model the minimum wage is 63% of median wages for unskilled workers, but just 40% for skilled workers. Without a lower elasticity of substitution for unskilled workers than skilled, the unemployment gap between the two groups would be counter-facutally large.[11] Similarly, without a higher bargaining parameter for unskilled workers than skilled the gap between the minimum wage coverage for the two groups would be counter-factually high.[12]

While studies such as Cahuc et al. (2006) find bargaining power decreases with skill, the parameter plays a very different role in their estimation strategy than in mine. In Cahuc et al. (2006) the bargaining parameter is informative in matching model predictions regarding the wage distribution to the data, using their prior estimates of employer and employee fixed effects and job transition rates. In my

[11]If I raise the elasticity of substitution for unskilled workers to the level for skilled workers, their respective unemployment rates increase from 7.1% and 3.1% respectively in the model (which matches the data) to 17.8% and 3.6%.

[12]If I lower the bargaining parameter for unskilled workers to the level for skilled workers, their respective minimum wage coverage rates increase from 15.8% and 5.7% respectively in the model (and data) to 18.6% and 5.7%.

TABLE 3. Estimated Parameters

Parameter	Description	Source	Value
Ψ_u	Elasticity of substitution between unskilled workers	SMM Estimation	7.218
Ψ_s	Elasticity of substitution between skilled workers	SMM Estimation	28.875
μ	Share parameter determining skill premium in KORV production function	SMM Estimation	0.336
A	Total Factor Productivity	SMM Estimation	11.885
η_u	Variance parameter of worker ability distribution: unskilled workers	SMM Estimation	0.452
η_s	Variance parameter of worker ability distribution: skilled workers	SMM Estimation	0.494
ϕ_u	Nash Bargaining Parameter for unskilled workers	SMM Estimation	0.235
ϕ_s	Nash Bargaining Parameter for skilled workers	SMM Estimation	0.143
κ_u	Hiring cost: unskilled workers	SMM Estimation	308.889
κ_s	Hiring cost: skilled workers	SMM Estimation	1228.192

estimation, its primary impact, as discussed above, is to match model predictions of minimum wage coverage to the data, and makes use of employee data only.

The estimates of vacancy costs are perhaps more intuitive and suggest it is approximately 4.5 times more costly to post a vacancy for skilled workers than unskilled workers. Given vacancy posting costs reflect the flow value of all recruitment costs in the model, this differential appears qualitatively reasonable on the grounds that skilled workers are likely to require greater screening and on-the-job training.

4.3. *Non-targeted Empirical Moments: Macro Moments*

Table 4 compares the model's predictions regarding a range of macroeconomic moments to the data. I hit the labour share precisely, which is perhaps surprising given the parameters of the KORV production function were originally estimated in the context of a competitive labour market model, and in the U.S where the labour share has tended to be lower than the UK. Though far from conclusive, this suggests the model, at a macro level, features relatively strong levels of employer competition despite the presence of frictions. This impression is reinforced when I compare markups in the model to empirical estimates, as is done in the second row of Table 4. The mark-up measure I use comes from De Loecker and Eeckhout (2018), and is the ratio of output price to estimated marginal costs (so a perfectly competitive economy would have a mark-up ratio of 1).[13] My model

[13] De Loecker and Eeckhout (2018)'s estimator for markups at a given firm, υ_f is:

$$\upsilon_f = \varepsilon_f^I \frac{P_f^Q Q_f}{P_f^I I_f}$$

gives a mark-up measure that is significantly below the De Loecker and Eeckhout (2018) estimate for the UK (1.06 in the model vs an estimate of 1.5). This is likely to reflect two features of my model: (i) free entry in vacancy creation, which drives the expected profits from issuing a vacancy to zero; and (ii) when a worker is poached in the model, they receive a wage equal to their marginal product (i.e. the competitive wage). The model's mark-up prediction is not out of the range of the estimates given by De Loecker and Eeckhout (2018) however (e.g. it's consistent with mark-up estimates for the UK in the 1980s).

The third and final moment considered in Table 4 is the net capital to gross value added ratio, where the model suggests less capital intensity than observed in the data (1.77 in model vs 2.66 in data). Differences could partly reflect methodological differences in capital stock measurement in the data used to estimate the KORV production function and in the empirical moment so I do not attach too much importance to this discrepancy.

Table 4. Non-targeted Macro Moments

Moment	**Model Moment**	**Empirical Moment**
Labour Share of GVA[1]	0.76	0.76
Mark-Up Ratio[2]	1.06	1.5
Net Capital Stock/GVA[3]	1.77	2.6

[1] Bank of England, includes self-employed labour income (imputing it as compensation per employee multiplied by number of self-employed). GVA=Gross Value Added

[2] Empirical moment taken from De Loecker and Eeckhout (2018), model moment is calculated analagously (as described in text).

[3] UK National accounts, ONS.

Although I do target some moments of the wage distribution, it is nevertheless instructive to consider the fit of the entire model wage distribution to that in the data: see Figure 3. The model closely fits the empirical wage distribution except where the empirical distribution lies below the legal minimum wage, which likely reflects a mixture of measurement error and non-compliance.

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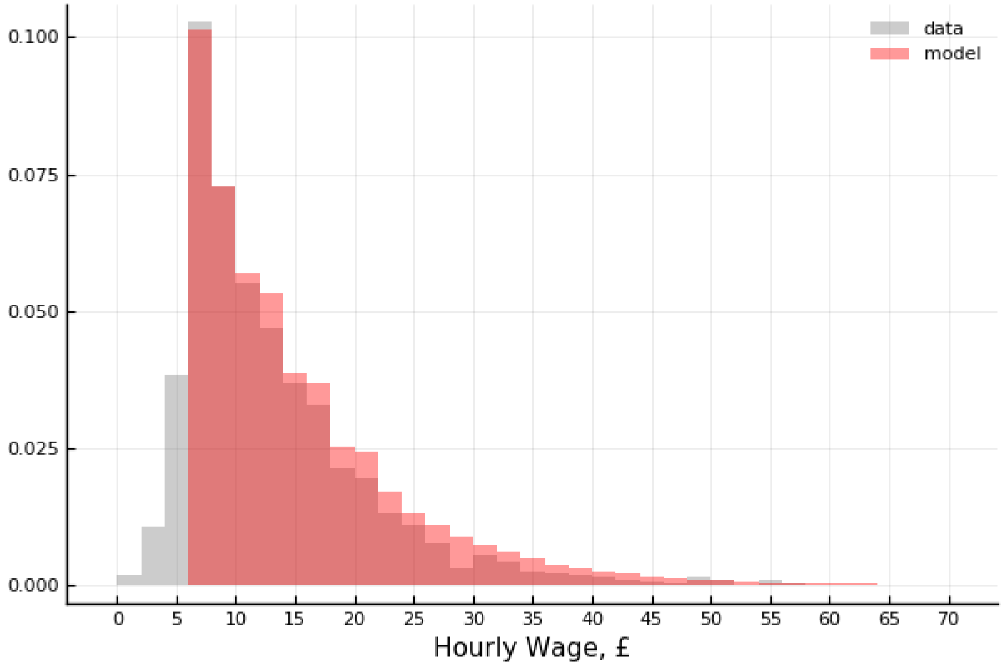
where I_f is the firms use of variable input I (with price denoted P_f^I), Q_f is their output (with price denoted P_f^Q) and ε_f^I is their output elasticity with respect to I_f. In my model, only intermediate firms have mark-ups, and have output elasticity of one due to constant returns to scale in intermediate good production. Constant returns makes the definition of an intermediate firm in principle ambiguous so I choose to define it as a collection of all firms employing a worker in sector j (where j again indexes the skill and ability of the workers employed in that sector). The model counterpart to De Loecker and Eeckhout (2018)'s mark up measure is therefore:

$$\upsilon_j = \frac{p_j(1 - e_j^{ue})\ell_j}{(\max(m_w, w_j^b)e_j^{np} + p_j e_j^p)\ell_j}$$

In both De Loecker and Eeckhout (2018) and my model counterpart, an average mark-up measure is calculated by taking the sales-weighted means of the firm mark-up measures shown above.

Figure 3. Model vs Empirical Wage Distribution



Notes: Data from LFS 2013-14, for workers who are paid by hour.

4.4. *Non-targeted Empirical Moments: Matching Reduced Form Evidence*

An important test of the model is whether it matches the reduced form evidence in the UK on minimum wage impacts. In this section, I examine whether the model can replicate the findings of Draca et al. (2011). They use a difference-in-difference methodology to estimate the impact of the introduction of the minimum wage in 1999 on firm profitability and average wages. Their treatment group is firms with average wages less than £12,000 in 1999. The average wage of this group is close to the level of the minimum wage. Their control group is firms with average wages between £12,000 and £20,000.

The only firms that earn profits in my model are intermediate goods firms, who sell competitively to final good firms but have some monopsony power over workers due to labour market frictions. The definition of a firm is in principle ambiguous due to constant returns to scale within the intermediate goods sector. I define the firm as a collection of all vacancies and active jobs in intermediate goods market j. The profit level, π_j, profit margin, π_j^m and average wage, $\bar{w}_j$ for a firm of type j are therefore as follows:

$$\pi_j = [(p_j - \max(w_j^b, m_w))e_j^{np} - v_j \kappa_j]\ell_j \tag{22}$$

$$\pi_j^m = \pi_j/(p_j(1 - e_j^{ue})\ell_j) \tag{23}$$

$$\bar{w}_j = (p_j e_j^p + \max(w_j^b, m_w)e_j^{np})/(1 - e_j^u) \tag{24}$$

Note that in the model the only variance in wages within a given firm comes from the proportion of workers that are poached or not, which can loosely be thought of as wage heterogeneity due to tenure.

As in Draca et al. (2011), I run the regressions in equations (25) through (28), where the subscript zero denotes the level of a variable before the minimum wage was introduced. I replicate the introduction of the minimum wage by first reestimating the set of parameters shown in Table 3 so that they match the models' predictions to the same empirical targets specified in section **??** but for 1998-99, i.e. before the minimum wage was introduced. I then simulate the steady-state impact of introducing the minimum wage at the level it was set at in April 1999, and running the regressions shown in equations 25 through to 28. I assume the economy is in steady state before and after the introduction of the minimum wage in my analysis, so the Δ in the regression equations represents the change in the dependent variable between steady states in the model. This is broadly consistent with the empirical exercise in Draca et al. (2011) which considers average profit and wage rates for the three years before and after the minimum wage introduction and thus also attempts to estimate a 'long-run' impact. [14]

$$\Delta\pi_j^m = const + \hat{\beta}_1 \; Treatment_j + \epsilon_j \tag{25}$$

$$\Delta\pi_j^m = const - \hat{\beta}_2 \; \log \bar{w}_{j0} + \epsilon_j \tag{26}$$

$$\Delta\bar{w}_j = const + \hat{\beta}_3 \; Treatment_j + \epsilon_j \tag{27}$$

$$\Delta\bar{w}_j = const - \hat{\beta}_4 \; \log \bar{w}_{j0} + \epsilon_j \tag{28}$$

The four regression coefficients $(\hat{\beta}_1, \hat{\beta}_2, \hat{\beta}_3, \hat{\beta}_4)$ and their standard errors are shown in Table 5, with their standard errors, both for simulations from the model and the original findings in Draca et al. (2011). The model comes close to replicating the average wage impact of the minimum wage, a result that is not mechanical given the degree of minimum wage spillover is endogenous to the model. While the model does not manage to replicate the absolute fall in profit margins, it almost exactly matches the % fall in profit margins (see third column of Table 5).

[14] The minimum wage did not significantly change in real terms in the three years after its introduction, either as a % of the median wage or in terms of consumer prices.

Table 5. Replicating Reduced Form Evidence

	Dependent Variable		
	Change in ln(average wage)	Abs Change in Profit Margin	% Change in Profit Margin
Results from Model:			
Dummy: Low Wage Firm	0.081	-0.0032	-22.81
	(0.0252)	(0.0017)	
-ln(initial average wage)	0.1939	-0.0074	
	(0.0266)	(0.0022)	
Results from Draca et al. (2011):			
Dummy: Low Wage Firm	0.09	-0.029	-22.66
	(0.026)	(0.012)	
-ln(initial average wage)	0.188	-0.032	
	(0.033)	(-0.015)	

While it is not the focus of their paper, Draca et al. (2011) also estimate the employment impact of the minimum wage introduction and "do not find any significant negative effects on employment", which is consistent with results elsewhere (e.g Leonard et al. (2014)). In contrast, the model predicts a significant increase in unemployment for low skilled workers (from 6.7% to 13.7%). This reflects the fact that the model captures only the disemployment impact of minimum wages and does not include other labour related margins of adjustment for firms like hours worked, or employee benefits (pensions, training etc). It also reinforces the earlier caveat that the predicted unemployment nonlinearities in the model should be viewed as a cautious/lower bound estimate of where any nonlinearities may be located in reality.

5. Results

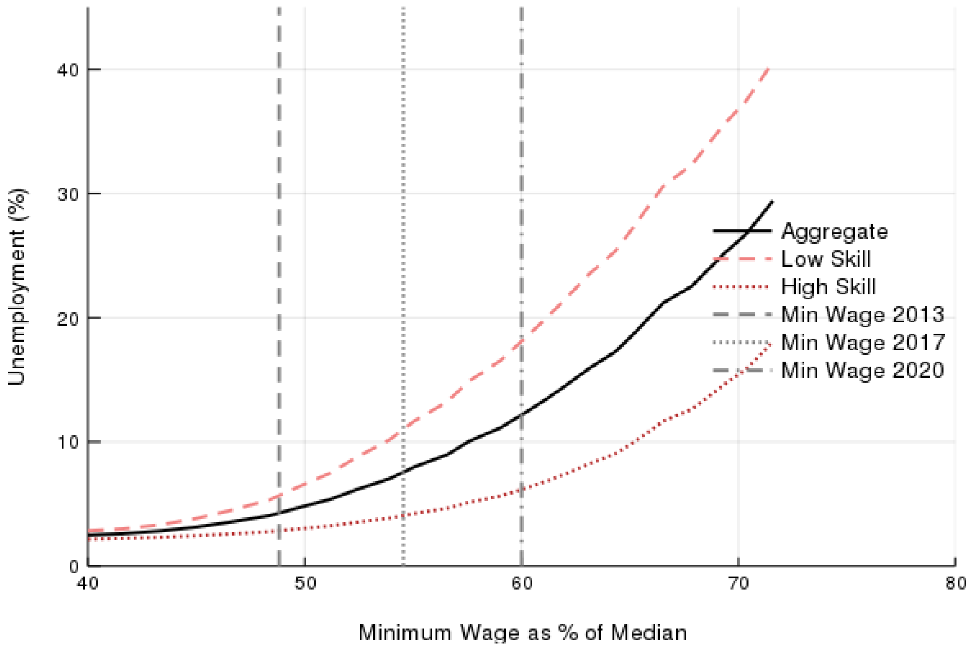
Figure 4 shows the simulated relationship between steady state unemployment in my model to the level of the minimum wage. All simulations shown in this section are steady state equilibrium outcomes, conforming to the equilibrium definition provided in Section 3.3, and so do not account for transition dynamics. The results suggest a significant risk of increased unemployment in the range of minimum wage values planned in the UK.

At first sight the unemployment response appears counter-factual; the model predicts that unemployment should have increased due to the minimum wage increases introduced from 2013 to the present date, but emprically this has not been the case.[15] However, inspecting headline movements in the unemployment data is not a substitute for econometric evaluation as there are likely to have been contemporaneous changes in the UK economy that might explain the fall in unemployment e.g. a cyclical improvement following the global financial crisis, and a structural

[15]the unemployment rate for those aged between 16 and 64 decreased from 7.8% in 2013 Q1 to 4.1% in 2018Q4 (source: Labour Force Survey)

decrease in unemployment due to a tightening of the welfare regime for unemployed workers. As far as I am aware, the only econometric research that considers both employment entry and exit impacts of the post 2013 increases in the minimum wage - Dickens and Lind (2018) - finds a small but significant decrease in employment, using variation in minimum wage bite by geographic area.

FIGURE 4. Nonlinear Unemployment Response



I now proceed to investigate what generates this nonlinear response in the model. I look at this question in two parts, first investigating the mechanisms that account for the existence of the nonlinearity, and then examining which parameters determine its location and strength.

The drivers of the nonlinear unemployment response in my model are: (i) endogenous nonlinearities in labour demand that arise both from using a multi-input production function with imperfect substitution between all inputs and from endogenous vacancy creation; and (ii) exogenous nonlinearities in the distribution of ability across workers, within a given skill type.

I investigate the quantitative importance of each of these factors by altering my baseline model in three ways. First I move to a final good production with labour as it's only input, and with perfect substitution between all skill and ability types.[16] This shuts off the endogenous nonlinearity in labour demand driven by imperfect

[16]The final good production function becomes $Y = \sum_{i=1}^{M} x_{u,i} y_{u,i} + \sum_{i=1}^{M} x_{s,i} y_{s,i}$, where as before $y_j = (1 - e_j^{ue})\ell_j$ but the ability level, x_j, of each worker type is set equal to the equilibrium

substitution between input types, but keeps the other sources of nonlinearities (endogenous vacancy creation and a non-uniform distribution of ability types). Second I shut off endogenous vacancy creation as a driver of a nonlinear unemployment response by no longer imposing the free entry condition 14.[17] The third change I make is to impose a uniform distribution of abilities and skills, rather than my baseline assumption of a log normal distribution of ability.[18] These channels are eliminated one at a time, rather than sequentially. Appendix C describes in more detail how I implement these alterations to my baseline model.

Figure 5 starts by shutting down the two endogenous sources of nonlinearities: vacancy creation and imperfect substitution between inputs. It suggests that, quantitatively, the impact of imperfect substitution is a much more significant driver of the nonlinear unemployment response than endogenous vacancy creation. Removing endogenous vacancy creation from the model yields almost exactly the same nonlinear relationship between the minimum wage and unemployment, whereas removing imperfect substitution between factor inputs yields a much more linear relationship. This is a significant implication of the model since existing search models of the minimum wage in the literature typically assume perfect substitution between inputs of production, and so are not able to capture this source of nonlinearity. When I look at the impact of a uniform distribution of skill types, as in Figure 6, I see that this change alone is enough to drive a largely linear response in unemployment, even in the presence of both endogenous nonlinearities, however the wage distribution in Figure 3 strongly suggests a non-uniform distribution of ability.

The picture that emerges from figures 5 and 6 is that, over the range of minimum wage values I consider, imperfect substitution between inputs is the most significant endogenous mechanism driving the nonlinear unemployment response. However, switching to a uniform distribution of ability types dominates the combined effect of both of the endogenous sources of nonlinearities. While this exercise is helpful in identifying which factors account for the existence of the nonlinearity, switching to completely uniform distribution of worker ability is clearly an extreme and likely unrealistic scenario: for example, the wage distribution shown in Figure 3 is suggestive of a non-uniform distribution of abilities that is approximately log normal.

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marginal product of the intermediate good produced by that worker type, $\frac{\partial Y}{\partial y_j(e_j^{ue^{ss}})}$, in my baseline model when the minimum wage is set to zero.

[17] I impose that job contact rates for the unemployed, $\lambda_{0,j}$, are initially set a fixed level equal to the equilibrium job contact rates in my baseline model, $\theta_j^{ss} q(\theta_j^{ss})$, when there is no minimum wage

[18] That is I assume $x \sim U(x_{min}, x_{max})$ where the boundaries of this interval are the same as under my baseline calibration.

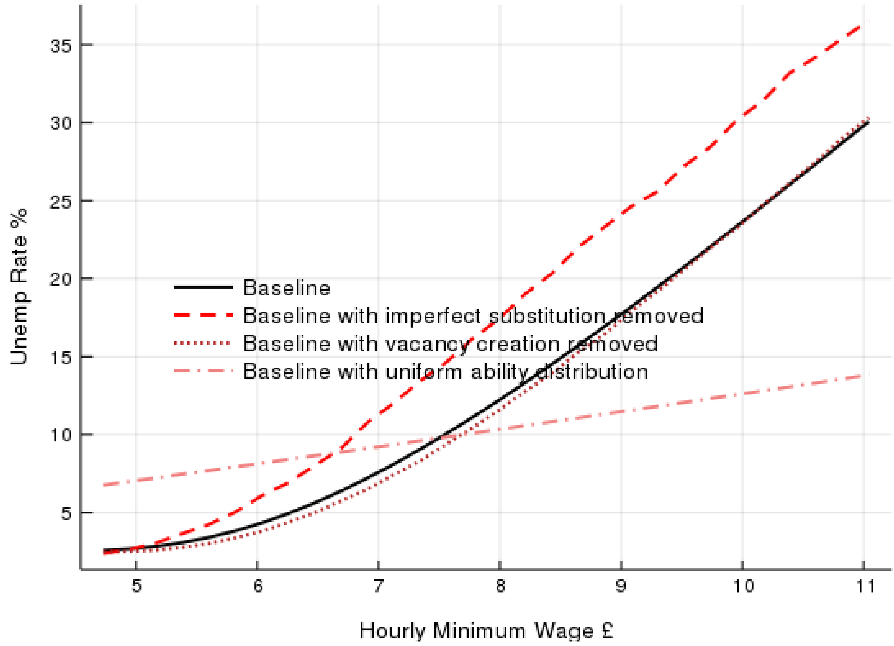
FIGURE 5. Sources of Nonlinearities: Endogenous

I now turn to the question of which parameters in my baseline model determine the strength of the nonlinear unemployment response. I do this by altering the parameter values used in my baseline calibration by plus and minus 25%. In each case I only alter one of the parameter values and leave the others unchanged.

The results are shown in Figure 7. The quantitative importance of the worker ability distribution again emerges; the first row of Figure 7 shows that the biting point of the nonlinearity occurs significantly later (i.e. at a higher minimum wage level) as I either decrease the dispersion of the unskilled workers' ability distribution, moving closer to a representative agent model, or by increasing the output share of the aggregate unskilled labour input (broadly equivalent to a rightward shift of the entire distribution of unskilled workers' ability).

I concluded above that the imperfect substitution of inputs in final good production was a more significant endogenous driver of the nonlinear unemployment response than vacancy creation. Figure 7 gives us the more specific conclusion that varying the elasticity of substitution between unskilled workers of differing abilities has a more significant impact on the location and strength of the nonlinearity than

Figure 6. Sources of Nonlinearities: All



varying the elasticity of substitution between the aggregate unskilled input, U, and capital equipment, K_{eq}.[19]

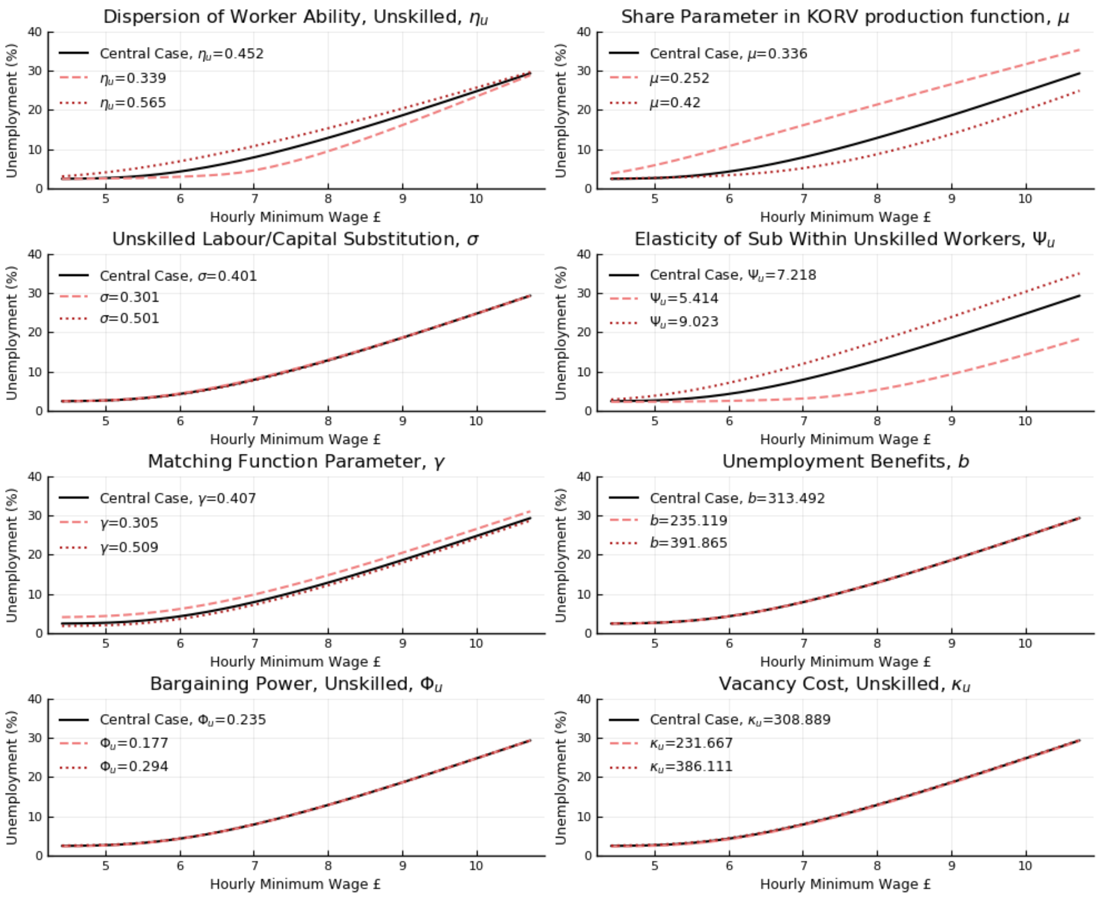
The remaining plots in Figure 7 reflect the quantitative lack of importance of endogenous vacancy creation in the model. The matching function parameter has a level impact on unemployment, but does not significantly change the unemployment response to the minimum wage. Varying the level of unemployment benefits, bargaining power and the cost of vacancy posting by plus and minus 25% all have negligible impacts on the strength and location of the nonlinear unemployment response. This is not particularly surprising given that all of these parameters have a direct impact on the vacancy creation channel only, which I have previously found to be a relatively unimportant source of the unemployment nonlinearity in the model.

6. Conclusion

This paper has examined whether there are likely to be significant nonlinearities in the impact of the minimum wage on unemployment. I explored this question using a model that combines search frictions with a production process featuring several

[19]The specification of the KORV production function implies the elasticity of substitution between the aggregate unskilled input and capital equipment always equals the elasticity of substitution between the aggregate unskilled and skilled intermediate inputs, U and S.

FIGURE 7. Sensitivity Analysis



margins of substitution between factor inputs. In this context, nonlinearities are driven by: (i) endogenous nonlinearities in labour demand that arise both from using a multi-input production function and from endogenous vacancy creation; and (ii) exogenous nonlinearities in the distribution of skill across workers. When calibrated to match the UK economy, the model suggests a nonlinear unemployment reaction that bites well within the range of minimum wage levels planned in the UK over the next two years.

Quantitative results from the model suggest that the most significant endogenous driver of a nonlinear relationship between the minimum wage and unemployment is the imperfect substitution between different worker ability types in the production function. If we instead assume constant returns to scale production using labour as the only input of production, as is commonly done in the search literature, the predicted relationship becomes significantly more linear. This highlights the importance of allowing for imperfect substitution between factor inputs when considering the unemployment impacts of the minimum wage.

I view the predictions of this model as a somewhat cautious lower bound estimate of where any nonlinearities might lie since the minimum wage can only ever have a negative impact on the employment rates of workers for whom the minimum wage binds in the model. There are plausible mechanisms that could break this result. The minimum wage could increase worker search effort or labour market participation, as in Flinn (2006), or could screen low productivity firms out of the market and so allow higher productivity firms to expand (Mayneris et al. (2014)). Including these mechanisms in the model developed in this paper is a worthwhile goal for further research, particularly as it could address the counterfactually large unemployment response in the model.

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Appendix A. Parameter Impacts on Model Moments

While the lack of closed form solutions in the model prevents proof of identification, it is nevertheless instructive to explore how varying the magnitude of the parameters I estimate affects the simulated model moments. I do this in Figure 8, which looks at the impact of varying each of the estimated parameters by plus and minus 25% from its value in my baseline estimation.

First, and somewhat reassuringly, none of the parameters *individually* appear to have observationally equivalent impacts on the model moments i.e. each produce a distinct range of impacts. Of course this does not imply identification, where the more relevant question is whether jointly varying a combination of parameters has observationally equivalent impacts on the model moments as varying any single parameter. Nonetheless, it is instructive to consider how my parameters individually effect the various model moments.

η_u and η_s are the parameters that determine the dispersion of the log normal ability distribution of unskilled and skilled workers respectively. As expected, they have positive monotone impacts on the model moments related to wage dispersion: the standard deviation of log wages, and the p90/50 and p50/10 ratios of the wage distributions. The dispersion parameters also affect median wages, albeit only weakly.

It is notable that increasing dispersion parameter for skilled workers increases the proportion of employees covered by the minimum wage , but not for unskilled workers. A priori this relationship is ambiguous: increase dispersion shifts mass from the centre of the wage distribution to the left and right tails, which means some workers who were paid the minimum wage go into unemployment, lowering the coverage rate, but also shifts mass from slightly higher up the wage distribution closer to the minimum wage, raising the coverage rate. For unskilled workers, the unemployment effect is relatively strong and offsets the inflow of somewhat higher paid workers into the minimum wage. For skilled worker, the latter effect is dominant so minimum wage coverage increases with the dispersion of ability.

The elasticity of substitution between heterogeneous workers of differing abilities also monotonically increases the unemployment rate for a given skill group, but monotonically decreases measures of wage dispersion. The unemployment impact is as expected: as workers become more substitutable, the presence of a fixed minimum wage (it is set at it's 2013-14 level in my baseline calibration) causes greater unemployment of low skilled workers. The intuition behind the decrease in wage dispersion is that as it becomes easier to substitute workers in production there is less of a premium for scarcity, which decreases wage dispersion.[20] The ease of substitution between workers within a given skill type also has strong positive impact on the proportion of these workers being paid the minimum wage for both skill groups.

[20]This relationship is also ambiguous a priori: increasing the elasticity of substitution decreases the scarcity premium both for scarce high ability workers, and scarce low ability workers. The latter impact could in theory be dominant and raise wage dispersion but does not in my calibration.

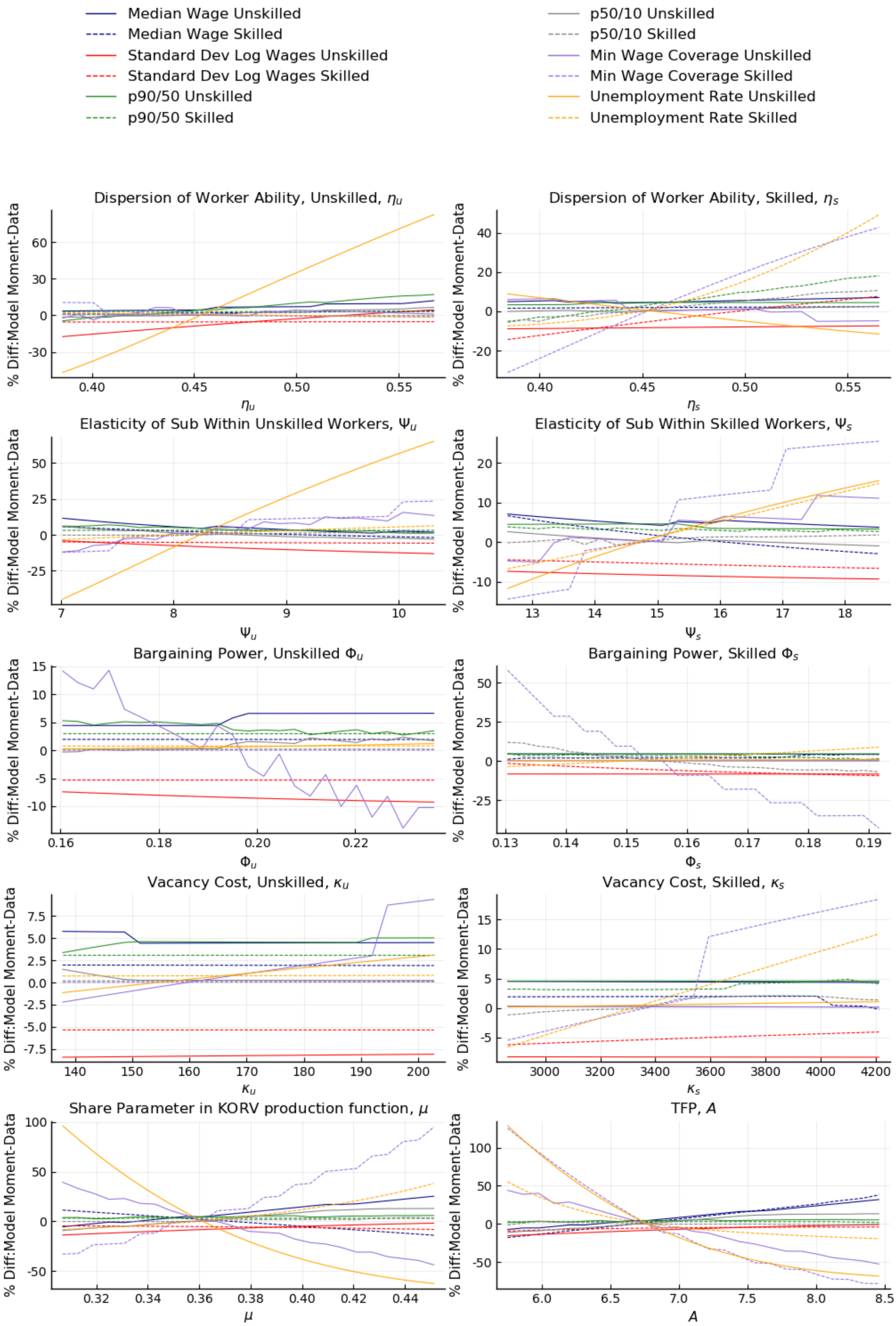
The bargaining parameter is the most directly relevant on the model predictions for minimum wage coverage, in the sense that, as expected, it has a relatively strong monotone negative effect on minimum wage coverage, but only a relatively weak impact on other moments.

Perhaps surprisingly, the cost of vacancy posting has a strong positive effect on minimum wage coverage and a relatively weaker impact on unemployment. This reflects the presence of on-the-job search in the model. Increasing the vacancy cost means lower job contact rates for unemployed and employed workers (job contact rates for employees are directly proportional to contact rates for unemployed workers by assumption). Since job contacts raise employees' wages, a reduction in the contact rate shifts the wage distribution to the left and so increases minimum wage coverage. The relatively weak impact of vacancy costs on unemployment reflects the fact that a substantial part of unemployment is caused by the impact of the minimum wage on demand for intermediate goods by final good producers and that the cost of vacancy creation does not have a significant impact on this relationship.

Finally the TFP and the share parameter, A and μ in the KORV production function have the expected impacts: TFP increases wages and employment for both unskilled and skilled workers, whereas μ, which determines the output share of the unskilled intermediate sectors, improves wage and employment outcomes for the unskilled at the expense of the skilled.

The assumption that ability is log normally distributed plays an important role in allowing me to use the chosen empirical moments to discipline my parameter estimates, especially with regard to using the minimum wage coverage rate to discipline the bargaining parameter, as discussed in Flinn and Heckman (1982). The model would likely be severely under-identified if we allowed the ability distribution to take a more flexible non-parametric form. However, the model's ability to match the wage distribution very closely - see Figure 3 - suggets the assumption of a log normal distribution is a reasonable one.

Figure 8. Parameter Impacts on Model Moments



Appendix B. Minimum Wage impacts: Wage Spillovers

I define minimum wage spillovers to mean any change in the shape of the wage distribution above the minimum wage, other than a purely mechanical truncation effect. [21]

As discussed above, the fact that the minimum wage acts as a side constraint on the Nash bargained wage rules out wage spillovers due to pure bargaining impacts. However, changes to labour demand in the model generate both *within* and *between* group spillovers, where group is defined both by the skill level of the worker and their heterogeneous ability type. I again index these groups by j.

The imposition of a minimum wage, m_w, generates within group spillovers for type j workers whenever the minimum wage is in the range $w_j^b < m_w < p_j$. In this scenario, not poached type j workers receive the minimum wage rather than their Nash bargained wage i.e. there is no spillover within a given employment state. The resulting wage increase means intermediate firms reduce their vacancy creation relative to a counterfactual scenario with no minimum wage, which increases the unemployment rate until the equilibrium condition shown in equation (20) again holds. This generates a wage spillover for poached workers who see their wage increase, since it equals the marginal product of the intermediate good they produce which rises as employment falls. Thus despite their wage initially exceeding the minimum wage, poached workers will still see their wage increase due to the imposition of the minimum wage i.e. there is a positive spillover. However, reduced vacancy creation decreases job-to-job mobility rates, so although the poached workers see their wages increase, the density of such workers decrease. The net impact on the within group wage distribution depends on the relative magnitude of the positive spillover from the increase in the poached wage and the negative spillover from reduced job mobility rates.

Between group spillovers are generated because raising the price of one type of worker via the minimum wage always alters demand for all other types. The direction of spillovers between workers of different skill and ability types will depend on the degree of complementarity in a given calibration of the production function.

Appendix C. Sources of Nonlinearities

I isolate the sources of the nonlinear unemployment response in my model by simulating results from three alternative models. Each of these alternative models has a different feature removed from the baseline model. The three features that cause the nonlinear unemployment response in my baseline model are imperfect substitution between factor inputs, endogenous vacancy creation and the non-uniform distribution of workers' abilities. Accordingly, the first of the three alternative models I discuss here has perfect substitution between factor inputs, where factor inputs are high and low skill labour of

[21] see e.g. Flinn (2002) - a mechanical truncation impact of the minimum wage occurs whenever the minimum wage decreases aggregate unemployment, and therefore increases the wage density for all remaining employed workers even if their employment levels are unchanged.

varying ability types. This alteration removes imperfect substitution of factor inputs as a driver of the nonlinear unemployment response (but otherwise the model is as per my baseline model). The second model I discuss removes endogenous vacancy creation from the baseline model but keeps all other factors the same. Finally, I discuss a version of my model with a uniform distribution of worker abilities, again keeping all other features as per my baseline model. In each case, I will describe exactly how the change is implemented, how it affects the solution algorithm I use to solve for equilibrium in the model, and any necessary changes to calibrated parameters.

C.1. *Sources of Nonlinearities: Imperfect Substitution*

I consider the impact of allowing imperfect substitution by constructing an alternative model with perfect substitution between factor inputs. In this alternative model, the final good production function becomes $Y = \sum_{i=1}^{M} x_{u,i}y_{u,i} + \sum_{i=1}^{M} x_{s,i}y_{s,i}$, where as before $y_j = (1 - e_j^{ue})\ell_j$. The only price consistent with non-zero equilibrium employment rate in the intermediate goods sector is $p_j = x_j$. If the price is above this point, final good producers will not demand any y_j. If $p_j < x_j$ final good producers demand an infinite amount of y_j, which is not consistent with equilibrium as this implies zero unemployment at which point recruitment costs for intermediate firms are infinite. In this environment, which is the standard production function assumed in most search models, minimum wages have a cliff-edge impact: when m_w exceeds x_j, employment of that type falls to zero.

The solution algorithm I use to solve this alternative model is as follows (italicised text emphasises differences from our baseline algorithm):

(1) Guess the unemployment rate e_{j0}^{ue}, $\forall j \in \{(u,1)..(u,M),(s,1)...(s,M)\}$.

(2) Set $p_j = max(m_w, x_j)$. *Unlike in baseline model, p_j is now independent of the unemployment guess e_{j0}^{ue}*

(3) Use the conditions (16) and (17) to derive vacancy levels necessary for the unemployment guess $e_{j,0}^{ue}$ to be consistent with steady state in the labour market. This then implies employment transition probabilities for the unemployed and employed via the matching function: $\theta_j q(\theta_j)$ and $\chi_j \theta_j q(\theta_j)$ respectively.

(4) Use employment transition probabilities from above and condition that poached worker is paid $w_j^p = max(p_j, m_w)$ to solve worker value functions and Nash bargained wage using equations (3) to (5) and (15) respectively. Wage of not-poached worker is whatever is highest of this bargained wage and minimum wage.

(5) Update employment guess:

   (a) *If $m_w > x_j$, then set $e_{j1}^{ue} = 1$. Note in baseline model, $e_j^{ue} = 1$ is not consistent with equilibrium as the intermediate good has infinite marginal product at zero.*

(b) If $m_w <= x_j$, use wage levels from above steps to give an updated unemployment guess, e^{ue}_{j1}, $\forall j \in \{(u,1)..(u,M),(s,1)...(s,M)\}$ that simultaneously solves free entry condition (14) for the intermediate firm and the final good firm's FOC i.e. equations (18) and $p_j = x_j$.

(6) Repeat iteration until convergence of unemployment guess.

In the simulations presented in Figures 5 and 6 for this alternative model, I use exactly the same parameters as under my baseline calibration with one exception. I impose that the ability levels x_j in this alternative model are exogenously set at the price of intermediate goods in the baseline model when there is no minimum wage. This ensures that in the absence of the minimum wage, this alternative model predicts the same wage and unemployment levels as in the baseline model.

C.2. *Sources of Nonlinearities: Endogenous Vacancy Creation*

In my baseline model, intermediate firms respond to a binding increase in the minimum wage by reducing vacancy creation. Remaining vacancies are filled at a higher rate, which reduces recruitment costs (and increases the price of the intermediate good) until the point where the expected profits from issuing a vacancy are again zero. In the alternative model considered here, I effectively assume the supply of vacancies is completely inelastic so that, in the absence of a minimum wage, contact rates for unemployed and not-poached workers are fixed at a level $\lambda^*_{0,j}$ and $\lambda^*_{1,j}(=\chi_j\lambda^*_{0,j})$ respectively which imply an unemployment rate e^{ue*}_j (I will describe how I calibrate $\lambda^*_{0,j}$ shortly). When the minimum wage is imposed then, if $p_j(=\partial Y/\partial y_j(e^{ue*}_j)) \geqslant m_w$ then the unemployment rate and contact rates are unchanged. If $p_j(=\partial Y/\partial y_j(e^{ue*}_j)) < m_w$ then the unemployment rate and contact rates exogenously adjust until the marginal product of the intermediate good is raised to the level of the minimum wage. This model therefore does not include the fall in recruitment costs from endogenous vacancy creation as a force restoring the intermediate goods market to equilibrium following a minimum wage change.

The solution algorithm I use to solve this alternative model is as follows (italicised text emphasises differences from our baseline algorithm).

(1) Guess the unemployment rate $e^{ue}_{j0} = e^{ue*}_j$, $\forall j \in \{(u,1)..(u,M),(s,1)...(s,M)\}$.

(2) Use this guess to construct the aggregate output of intermediate goods produced in the unskilled and skilled intermediate sectors (these aggregate outputs, U and S, are defined in equation (2)).

(3) Solve the final good firm's FOCs (equations (9) and (10)) to get their optimal choice of capital equipment and structures, K_{eq} and K_{st} that is consistent with the implied levels of U and S from above and firm optimisation. Then derive the price of each intermediate good p_j that is consistent with firm optimisation at the unemployment guess $e^{ue}_{j,0}$ using the FOCs in equations (7) and (8).

(4) Use the steady state condition that $\delta_j(1 - e_j^{ue}) = \lambda_{0,j} e_j^{ue}$, to derive job contact rates for the unemployed and not-poached workers as a function of the unemployment guess e_{j0}^{ue}.

(5) Use job contact rates from above and condition that poached worker is paid $w_j^p = max(p_j, m_w)$ to solve worker value functions and Nash bargained wage using equations (3) to (5) and (15) respectively. Wage of not-poached worker is whatever is highest of this bargained wage and minimum wage.

(6) Update employment guess:

  (a) *If* $m_w > p_j$*, then set* e_{j1}^{ue} *at the level that equates* $p_j(= \partial Y/\partial y_j(e_j^{ue}))$ *with* m_w.

  (b) *If* $m_w <= p_j$*, then* $e_{j1}^{ue} = e_{j0}^{ue}$.

(7) Repeat iteration until convergence of unemployment guess.

In the simulations presented in Figures 5 and 6 for this alternative model, I use exactly the same parameters as under my baseline calibration with one exception. I impose that the contact rates for the unemployed and employed, $\lambda_{0,j}^*$ and $\lambda_{1,j}^*$, equal the endogenously determined contact rates in the baseline model when there is no minimum wage. This ensures that in the absence of the minimum wage, this alternative model predicts the same wage and unemployment levels as in the baseline model.

C.3. *Sources of Nonlinearities: Non-uniform distribution of ability types*

In this alternative model, I do not alter any of the fundamental mechanisms of the baseline model but simply impose that $x \sim U(x_{min}, x_{max})$ where the boundaries of this interval are the same as under my baseline calibration, which are the same for unskilled and skilled workers. The equilibrium definition and solution algorithm remain as in the main body of this paper.