Minimum Wages in the UK
Searching for Non-linearities

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Minimum wages are an increasingly popular policy response to low wage growth for low paid workers.

**Figure: Minimum wages on the rise**
Politcal logic behind minimum wage (MW) increases in UK seems to be: “introduction of MW doesn’t seem to have hit unemployment, so let’s put it up some more”

This is risky: is it a linear relationship or are there nonlinearities to be wary of?

To answer this, we need a model to forecast impacts.

The model presented here can, eventually, help to address this and a wide range of questions:

1. Are there significant nonlinearities in minimum wage impacts?
2. How does minimum wage compare to other redistributive policies?
3. What are likely long term impacts on e.g. productivity, capital use, income and wealth inequality?
Preview of Results

- We develop a model that combines search frictions with a production function featuring several margins of substitution between factor inputs.

- Nonlinear unemployment reaction in model from:
  1. **Exogenous nonlinearities:**
     - Non-uniform distribution of skills.
  2. **Endogenous nonlinearities:**
     - Vacancy creation with Cobb-Douglas matching function
     - Imperfect substitution between capital and labour and between labour types

- When calibrated to the UK economy, we find:
  1. quantitatively, imperfect substitution between inputs is most important endogenous source of nonlinearities
  2. nonlinearity in unemployment lies within range of minimum wages planned in UK over next two years.
Outline

1. Introduction
2. Model
3. Calibration
4. Results
5. Conclusion
Methodology: Key Ingredients

1. **Frictional Labour Markets.** Search frictions can help explain findings of small impacts of UK minimum wage on employment and firm exit.

2. **Capital.** How does the minimum wage affect firms’ choice of capital vs. labour?

3. **Heterogeneous Agents.** Will minimum wage hikes cause companies to substitute towards higher skill workers?
Methodology: Ingredients Missing...

1. **Hours Worked.** Labour is entirely discrete, but model could be extended to include hours worked, as chosen by firms and workers.

2. **Participation Margin.** We do not consider positive impact of minimum wage on labour market participation. Again useful extension.

3. **Firm Heterogeneity.** All firms use same technology in this model. But could there be a useful role for minimum wage in eliminating low productivity firms?

4. **Business Cycles.** Should minimum wage increase in recession to provide stimulus to high MPC workers, or decrease to support labour demand?
Related Literature

1. **Structural literature on optimum minimum wage**
   - Search with wage posting - van den Berg and Ridder (1998): no unemployment effects until minimum wage equals productivity level then match is destroyed
   - Search with wage bargaining - Flinn (2006): if vacancy creation is present then smooth unemployment response until minimum wage equals productivity level then match is destroyed
   - **Contribution**: Introduction of decreasing returns to labour in search frameworks, removes cliff-edge effects.

2. **Empirical literature on UK minimum wage**
   Small employment effects, decrease in firm profits and limited price effects e.g. Leonard et al (2014), Draca and Machin (2011).
   - **Contribution**: Developing a model consistent with these findings, but also capable of examining future risks.
The Model: Environment

Workers.

- Workers differ in observable skill level, which is given (not chosen).
- Two broad skill types - unskilled and skilled ($u$ and $s$).
- Within broad skill types workers, workers differ by unobservable productivity level.
- Unobservable productivity, indexed by $i$, of a skilled (unskilled) worker is denoted $x_{s,i}$ ($x_{u,i}$), for $i = 1..M$
- Productivity is distributed exogenously according to the pdf $l_s(x_{s,i})$ ($l_u(x_{u,i})$)
- Both workers and firms have a common discount factor, $\beta$ and are risk neutral
The Model: Environment

Firms

- We wish to allow for both capital to labour substitution in production and substitution between skill types.
- Not easy in pure search/match framework.
- Proposed solution is to have two sectors of production:
  1. **Intermediate sector with search frictions.**
     Intermediate firms hire labour and sell it onto a final good producer - think of hiring agencies.
     - One segmented intermediate sector for each skill and productivity level of workers.
  2. **Final good sector** that combines labour hired in intermediate sector and capital, with no frictions.
The Model: Environment

**Figure:** Model Economy Overview

- Skilled Labour
- Hiring Agencies for Unskilled
- Search Frictions (Sequential Auction)
- Unskilled Labour
- Hiring Agencies for Skilled
- Final Good Producer (KORV)
- Open Economy Capital Markets
The Model: Environment

Final Good Firms

- Competitive firms which produce using technology shown below. Inputs used:
  - $K_{eq}$ is amount of capital equipment, $K_{st}$ is amount of capital structures
  - $U$ is effective amount of goods purchased from the low skill intermediate sectors, $S$ is total effective labour from high skill intermediate sectors

\[
Y = AK_{st}^\alpha [\mu U^\sigma + (1-\mu)(\lambda K_{eq}^\rho + (1-\lambda)S^\rho)\frac{\sigma}{\rho}]^{\frac{1-\alpha}{\sigma}} \tag{1}
\]

\[
U = \left( \sum_{i=1}^{M} (x_{i,u}h_{i,u}) \frac{\Psi_u - 1}{\Psi_u} \right)^{\frac{\Psi_u}{\Psi_u - 1}}, S = \left( \sum_{i=1}^{M} (x_{i,s}h_{i,s}) \frac{\Psi_s - 1}{\Psi_s} \right)^{\frac{\Psi_s}{\Psi_s - 1}} \tag{2}
\]
The Model: Environment

Intermediate Firms

Notation: $j$ will be a vector valued index containing both the broad skill index $(u, s)$ and productivity index $(1..M)$ of a worker.

- One intermediate sector for each worker type $j$.
- One intermediate firm for every worker (so density of intermediate firms = density of workers)
- Number of matches given by matching function $M(S_j, V_j)$. $S_j =$ number of effective type m job searchers. $V_j =$ vacancies.
- $\theta_j \equiv V_j / S_j$ denotes labour market tightness
- Contact rate for type j firms is $q(\theta_j) \equiv M(S_j, V_j) / V_j$, and $(\theta_j q(\theta_j), \chi \theta_j q(\theta_j))$ are the contact rates for unemployed and employed workers respectively.
- Vacancies determined by free entry: i.e. firms issue a vacancy until expected profit equals vacancy cost.
The Model: Environment

Intermediate Firms: Wage Setting

- Assume that firms and unemployed workers engage in Nash bargaining - the minimum wage acts as a constraint to the Nash maximisation.

- When workers gets poached, incumbent and rival bid-up the wage until the value of employing a poached worker equals the value of carrying a vacancy i.e. zero (Postel-Vinay and Robin (2002))

- Therefore poached workers will get paid their marginal product in final good production.

- Minimum Wage reduces expected profit from employing not-poached worker, and decreases vacancy creation
The Model: Behaviour

Workers

A worker of a given type $j$ exist in one of three states:

- unemployed, receiving flow income $b$, with lifetime value function denoted $V_{j}^{ue}$

- employed but not poached, receiving the higher of Nash bargained wage $w_{j}^{b}$ and the minimum wage $m_{w}$, with value function $V_{j}^{np}$;

- employed and poached, receiving wage $w_{j}^{p}$, with value function $V_{j}^{p}$

$$V_{j}^{ue} = b + \beta[\theta_{j}q(\theta_{j})V_{j}^{np} + (1 - \theta_{j}q(\theta_{j}))V_{j}^{ue}]$$  \hspace{1cm} (3)

$$V_{j}^{np} = \max(w_{j}^{b}, m_{w}) + \beta[\delta_{j}V_{j}^{ue} + (1 - \delta_{j})[\chi\theta_{j}q(\theta_{j})V_{j}^{p} + (1 - \chi\theta_{j}q(\theta_{j}))V_{j}^{np}]]$$ \hspace{1cm} (4)

$$V_{j}^{p} = w_{j}^{p} + \beta[\delta_{j}V_{j}^{ue} + (1 - \delta_{j})V_{j}^{p}]$$ \hspace{1cm} (5)
Final Good Producers

- The firm’s profit maximisation problem is:

\[
\max_{K_{st}, K_{eq}, h_{i,u}, h_{i,s}, \forall i \in 1..M} \Pi = AK_{st}^\alpha [\mu U^\sigma + (1 - \mu)(\lambda K_{eq}^\rho + (1 - \lambda)S^\rho)^{\frac{\sigma}{\rho}}]^{\frac{1-\alpha}{\sigma}}
- \sum_{i=1}^{M} p_{i,u} h_{i,u} - \sum_{i=1}^{M} p_{i,s} h_{i,s} - r_{st} K_{st} - r_{eq} K_{eq}
\]

\[
U = \left( \sum_{i=1}^{M} (x_{i,u} h_{i,u})^{\frac{\Psi_u}{\Psi_u - 1}} \right)^{\frac{\Psi_u}{\Psi_u - 1}},
S = \left( \sum_{i=1}^{M} (x_{i,s} h_{i,s})^{\frac{\Psi_s}{\Psi_s - 1}} \right)^{\frac{\Psi_s}{\Psi_s - 1}}
\]

- Since final good producer is assumed to be competitive, all inputs are chosen to equalise marginal product is with the price of input.
The Model: Behaviour

Intermediate Firms

- Exist in one of three states:
  - carrying a vacancy, with firm value denoted by $J^v_j$,
  - employing a not-poached worker, $J^{np}_j$, and
  - employing a poached worker, with value $J^p_j$.

\[ J^v_j = -\kappa_j + \beta [q(\theta_j)\{s^n p_j J^p_j + (1 - s^n) J^v_j\} + (1 - q(\theta_j)) J^v_j] \]  
\[ J^{np}_j = p_j - \max(w^b_j, m_w) + \beta \left[ (1 - \delta_j)\{\chi \theta_j q(\theta_j) J^v_j + (1 - \chi \theta_j q(\theta_j)) J^{np}_j\} + \delta J^v_j \right] \]  
\[ J^p_j = p_j - w^p_j + \beta [(1 - \delta_j) J^p_j + \delta J^v_j] \]
The Model: Behaviour

Intermediate Firms

- Free entry, so $J_j^v = 0$, and Betrand competition between employers implies $J_j^p = 0$ so $w_j^p = p_j$.
- From these we get no entry condition:

$$\kappa_j = \beta q(\theta_j) s_j^u \frac{p_j - \max(w_j^b, m_w)}{1 - \beta(1 - \delta_j)(1 - \chi \theta_j q(\theta_j))} \tag{10}$$

- The bargained wage is given below ($\Phi$ is the nash bargaining parameter):

$$w_j^b = \argmax_{w_j^b} (V_j^{np} - V_j^u) \Phi_j (J_j^{np})^{1-\Phi_j}$$

$$= \Phi_j p_j + (1 - \Phi_j) (V_j^u (1 - \beta) - \beta(1 - \delta_j) \chi \theta_j q(\theta_j) (V_j^p - V_j^u)) \tag{11}$$
The Model: Equilibrium

Equilibrium: a sketch

- Steady State in Labour Markets

\[ \delta_j (1 - e_j^{ue}) = \theta_j q(\theta_j) e_j^{ue} \quad (12) \]
\[ (\delta_j + \chi \theta_j q(\theta_j)) e_j^{np} = \theta_j q(\theta_j) e_j^{ue} \quad (13) \]

- Solving gives us steady state unemployment and labour market tightness: \( e_j^{ues}, \theta_j^{ss} \)

- Intermediate goods market market clearing:

\[ p_j^s = \max(w_j^b, m_w) + \frac{\kappa_j \left( 1 - (\beta(1 - \delta_j)(1 - \chi \theta_j^{ss} q(\theta_j^{ss}))) \right)}{\beta q(\theta_j^{ss}) s_j^u} \quad (14) \]
\[ p_j^d = \frac{\partial Y}{\partial h_j(e_j^{ues})} \quad (15) \]
The Model: Minimum Wage Impacts

- From equilibrium conditions:

\[
\max(w^b_j, m_w) = \frac{\partial Y}{\partial h_j(e_j^{uest})} - \frac{\kappa_j \left(1 - (\beta(1 - \delta_j)(1 - \chi\theta^{ss}_j q(\theta^{ss}_j)))\right)}{\beta q(\theta^{ss}_j) s_j^u}
\]

(16)

- So wages = marginal product of labour minus recruitment costs

- Minimum wage increase implies:
  - intermediate firms to decrease vacancies. CD matching function: probability of filling remaining vacancies increases reducing recruitment cost.
  - reducing vacancies decreases employment, increasing marginal product of labour.
Calibration Approach

- Standard(ish) macro story: borrow some parameters from literature, estimate others (by SMM).
- We focus on estimating parameters for:
  1. exogenous distributions of worker productivity (log normal), with separate distributions for unskilled and skilled.
  - Empirical Targets: Variance of Log Wages and p90-10 ratios
  2. the elasticities of substitution between workers within these two skill classes, $\psi_u, \psi_s$.
  - Empirical Targets: Unemployment Rates
  3. recruitment costs $\kappa_u, \kappa_s$
  - Empirical Targets: Unemployment Rates
  4. the share parameter, $\mu$, in the KORV production function.
  - Empirical Targets: Graduate Wage Premium
Calibration Approach: Detail

- Denote the parameters to be estimated as \( \Phi = (\psi_u, \psi_s, \kappa_u, \kappa_s, A, \sigma_{u,x}, \sigma_{s,x}, \mu) \).
- Remaining parameters are taken from the literate, data or legislation and are denoted by \( \Omega \).
- Estimate the parameters in \( \Phi \) by SMM, targeting the following empirical moments for unskilled and skilled:
  - median wages,
  - variance of log wages,
  - \( p_{90}/10 \) and \( p_{50}/10 \) ratios.
  - unemployment rates.
- Let \( \hat{M} \) denotes vector of the empirical moments above, and \( \hat{M}(\Phi, \Omega) \) denote the model predictions of these moments. Then:
  \[
  \Phi^{SMM} = \arg\min_{\Phi} (M(\Phi, \Omega) - \hat{M})' (M(\Phi, \Omega) - \hat{M})
  \]  
  \[
  (17)
  \]
## Calibrated Parameters

### Table: Estimation Results

<table>
<thead>
<tr>
<th>Moment</th>
<th>Model Moment</th>
<th>Empirical Moment</th>
<th>% Deviation (Model - Data)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Median Hourly Wage: Unskilled</td>
<td>9.93</td>
<td>9.5</td>
<td>4.44</td>
</tr>
<tr>
<td>Median Hourly Wage: Skilled</td>
<td>16.01</td>
<td>15.71</td>
<td>1.94</td>
</tr>
<tr>
<td>Var Log Wages: Unskilled</td>
<td>0.45</td>
<td>0.49</td>
<td>-8.29</td>
</tr>
<tr>
<td>Var Log Wages: Skilled</td>
<td>0.54</td>
<td>0.57</td>
<td>-5.35</td>
</tr>
<tr>
<td>p90/50 Wages: Unskilled</td>
<td>2.01</td>
<td>1.92</td>
<td>4.57</td>
</tr>
<tr>
<td>p90/50 Wages: Skilled</td>
<td>2.02</td>
<td>1.96</td>
<td>3.12</td>
</tr>
<tr>
<td>p50/10 Wages: Unskilled</td>
<td>1.57</td>
<td>1.57</td>
<td>0.24</td>
</tr>
<tr>
<td>p50/10 Wages: Skilled</td>
<td>2.07</td>
<td>2.07</td>
<td>0.19</td>
</tr>
<tr>
<td>Min Wage Coverage: Unskilled</td>
<td>0.16</td>
<td>0.16</td>
<td>0.25</td>
</tr>
<tr>
<td>Min Wage Coverage: Skilled</td>
<td>0.06</td>
<td>0.06</td>
<td>0.03</td>
</tr>
<tr>
<td>Unemployment: Unskilled</td>
<td>0.07</td>
<td>0.07</td>
<td>0.51</td>
</tr>
<tr>
<td>Unemployment: Skilled</td>
<td>0.03</td>
<td>0.03</td>
<td>0.76</td>
</tr>
</tbody>
</table>
Calibrated Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Source</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Psi_u$</td>
<td>Elasticity of substitution between unskilled workers</td>
<td>SMM Estimation</td>
<td>8.251</td>
</tr>
<tr>
<td>$\Psi_s$</td>
<td>Elasticity of substitution between skilled workers</td>
<td>SMM Estimation</td>
<td>14.833</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Share parameter determining skill premium in KORV production function</td>
<td>SMM Estimation</td>
<td>0.361</td>
</tr>
<tr>
<td>$A$</td>
<td>Total Factor Productivity</td>
<td>SMM Estimation</td>
<td>6.765</td>
</tr>
<tr>
<td>$\eta_u$</td>
<td>Variance parameter of worker ability distribution: unskilled workers</td>
<td>SMM Estimation</td>
<td>0.454</td>
</tr>
<tr>
<td>$\eta_s$</td>
<td>Variance parameter of worker ability distribution: skilled workers</td>
<td>SMM Estimation</td>
<td>0.452</td>
</tr>
<tr>
<td>$\phi_u$</td>
<td>Nash Bargaining Parameter for unskilled workers</td>
<td>SMM Estimation</td>
<td>0.189</td>
</tr>
<tr>
<td>$\phi_s$</td>
<td>Nash Bargaining Parameter for skilled workers</td>
<td>SMM Estimation</td>
<td>0.153</td>
</tr>
<tr>
<td>$\kappa_u$</td>
<td>Hiring cost: unskilled workers</td>
<td>SMM Estimation</td>
<td>162.182</td>
</tr>
<tr>
<td>$\kappa_s$</td>
<td>Hiring cost: skilled workers</td>
<td>SMM Estimation</td>
<td>3369.239</td>
</tr>
</tbody>
</table>
### Table: Calibrated Parameters

<table>
<thead>
<tr>
<th>Parameter (Subscript)</th>
<th>Description</th>
<th>Source</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_u$</td>
<td>Job destruction rate: unskilled</td>
<td>LFS 2013q4-2014q3</td>
<td>0.011</td>
</tr>
<tr>
<td>$\delta_s$</td>
<td>Job destruction rate: skilled</td>
<td>LFS 2013q4-2014q3</td>
<td>0.007</td>
</tr>
<tr>
<td>$\chi_u$</td>
<td>Relative search intensity of employed to unemployed: unskilled</td>
<td>LFS 2013q4-2014q3 (ratio of employer change rate to unemployment exit)</td>
<td>0.112</td>
</tr>
<tr>
<td>$\chi_s$</td>
<td>Relative search intensity of employed to unemployed: skilled</td>
<td>LFS 2013q4-2014q3 (ratio of employer change rate to unemployment exit)</td>
<td>0.075</td>
</tr>
<tr>
<td>$b$</td>
<td>Monthly Unemployment benefits (job seekers allowance)</td>
<td>Legislative level 2013-14</td>
<td>313.492</td>
</tr>
<tr>
<td>$m_w$</td>
<td>Hourly minimum wage</td>
<td>Legislative level 2013-14</td>
<td>6.31</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Elasticity of substitution between unskilled and skilled workers</td>
<td>Krusell et al. (2000)</td>
<td>0.401</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Elasticity of substitution between skilled workers and capital equipment</td>
<td>Krusell et al. (2000)</td>
<td>-0.495</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Capital Structures Parameter</td>
<td>Krusell et al. (2000)</td>
<td>0.117</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Input share parameter for capital equipment and skilled labour</td>
<td>Krusell et al. (2000)</td>
<td>0.3</td>
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<tr>
<td>$\gamma$</td>
<td>Matching Parameter</td>
<td>Hagedorn and Manovskii</td>
<td>0.407</td>
</tr>
</tbody>
</table>
## Results: Matching Reduced Form Evidence

### Table: Replicating Reduced Form Evidence

<table>
<thead>
<tr>
<th></th>
<th>Change in ln(average wage)</th>
<th>Abs Change in Profit Margin</th>
<th>% Change in Profit Margin</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Results from Model:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dummy: Low Wage Firm</td>
<td>0.081</td>
<td>-0.003</td>
<td>-18.3</td>
</tr>
<tr>
<td>-ln(initial average wage)</td>
<td>0.1899</td>
<td>-0.0069</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0147)</td>
<td>(0.0005)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0156)</td>
<td>(0.0005)</td>
<td></td>
</tr>
<tr>
<td><strong>Results from Draca et al. (2011):</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dummy: Low Wage Firm</td>
<td>0.09</td>
<td>-0.029</td>
<td>-22.66</td>
</tr>
<tr>
<td>-ln(initial average wage)</td>
<td>0.188</td>
<td>-0.032</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.026)</td>
<td>(0.012)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.033)</td>
<td>(-0.015)</td>
<td></td>
</tr>
</tbody>
</table>
Results: Searching for Nonlinearities
Results: Drivers of Nonlinearities
Conclusions and Next Steps

- We develop a model that combines search frictions with a production function featuring several margins of substitution between factor inputs.

- Nonlinear unemployment reaction in model from:
  1. **Exogenous nonlinearities**:
     - Non-uniform distribution of skills.
  2. **Endogenous nonlinearities**:
     - Vacancy creation with Cobb-Douglas matching function
     - Imperfect substitution between capital and labour and between labour types

- When calibrated to the UK economy, we find:
  1. quantitatively, imperfect substitution between inputs is most important endogenous source of nonlinearities
  2. nonlinearity in unemployment lies within range of minimum wages planned in UK over next two years.