Minimum Wages in the UK

Searching for Non-linearities

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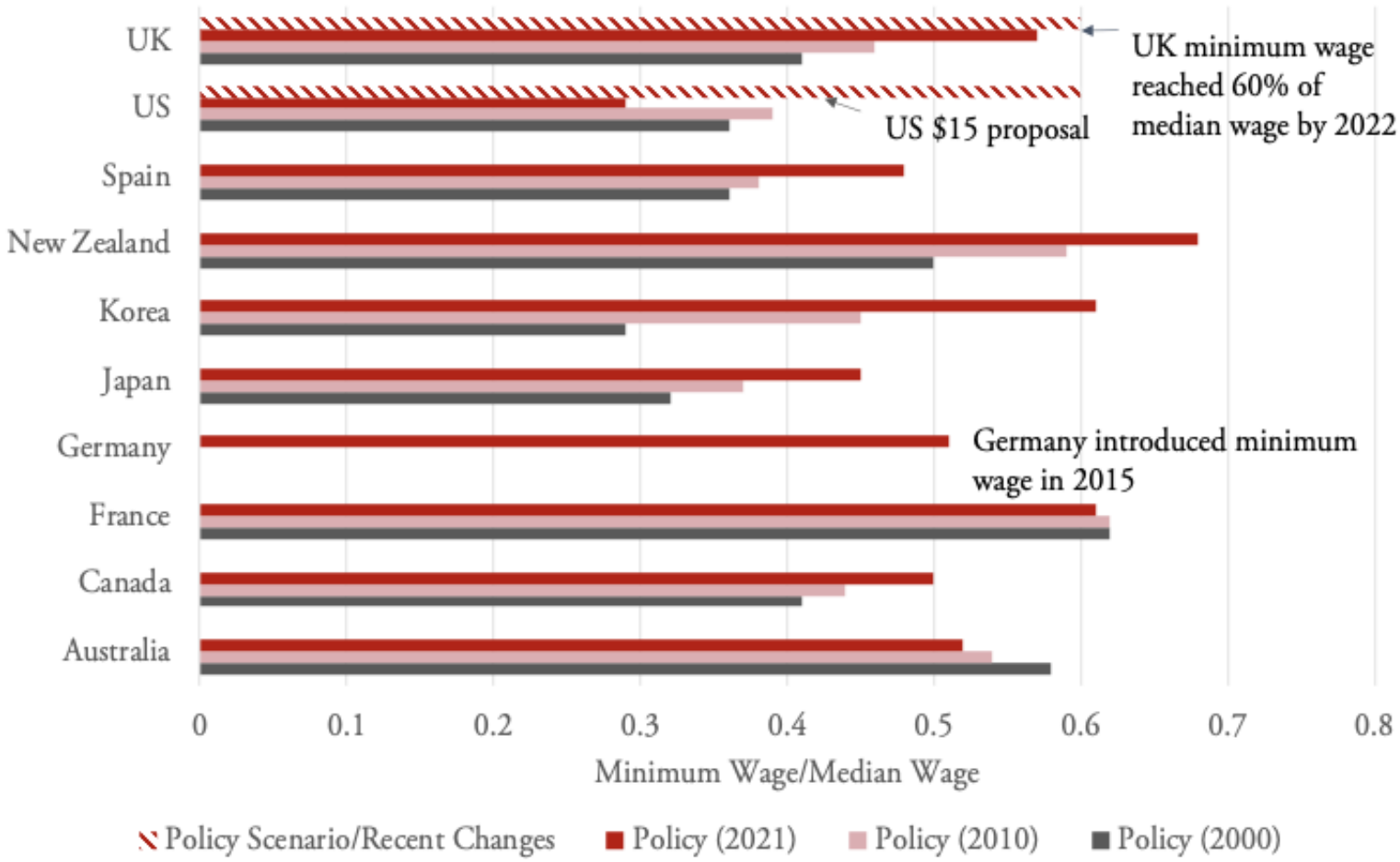
Motivation

- Minimum wages are an increasingly popular policy response to low wage growth for low paid workers.

Figure: Minimum wages on the rise



Source: OECD and Own Calculations

Motivation

- Political logic behind minimum wage (MW) increases in UK seems to be: "introduction of MW doesn't seem to have hit unemployment, so let's put it up some more"
- This is risky: is it a linear relationship or are there **nonlinearities** to be wary of?
- To answer this, **we need a model** to forecast impacts.
- Key research questions for model to address:
  1. Can it replicate past empirical findings regarding wage,profit and employment impacts of minimum wage?
  2. Are there significant nonlinearities in unemployment impacts of minimum wage?

Preview of Results

- **Theory Contribution.** I develop a model that combines search frictions (including on-the-job search) with a production function featuring imperfect substitution between factor inputs.
- Nonlinear unemployment reaction in model from:
  1. **Exogenous nonlinearities**:
     - Non-uniform distribution of skills.
  2. **Endogenous nonlinearities**:
     - Vacancy creation with Cobb-Douglas matching function
     - Imperfect substitution between factor inputs
- **Quantitative Contribution.** When calibrated to the UK economy, I find:
  1. quantitatively, imperfect substitution between inputs is an important endogenous source of nonlinearities
  2. nonlinear unemployment reaction of unskilled workers starts to bite (gently) when minimum wage is around 55-60% of median wage.

Outline

1 Introduction

2 Model

3 Calibration

4 Results

5 Conclusion

Methodology: Related Literature

1. **Search literature on optimum minimum wage.**
   - Wage posting - van den Berg and Ridder (1998): no unemployment effects until minimum wage equals ability level then match is destroyed.
   - Wage bargaining - Flinn (2006): endogenous vacancy creation means smooth unemployment response until minimum wage equals ability then match is destroyed
   - *Contribution: Introduction of decreasing returns to labour in search framework removes cliff-edge effects. Some precedent: Acemoglu (2001), Bauducco and Janiak (2018).*
2. **Empirical literature on UK minimum wage.**
   - Small employment effects, fall in firm profits and limited price effects Leonard et al (2014), Draca and Machin (2011).
   - *Contribution: Developing a model consistent with (some of) these findings, but also capable of examining future risks.*

The Model: Environment

Workers.

- Workers differ in observable skill level, which is ex-ante given.
- Two broad skill types - unskilled and skilled (u and s).
- Within broad skill types workers, workers differ by unobservable ability level.
- Unobservable ability, indexed by i, of a skilled (unskilled) worker is denoted $x_{s,i}$ ($x_{u,i}$), for $i = 1..M$
- Ability is distributed exogenously according to the pdf $l_s(x_{s,i})$ ($l_u(x_{u,i})$)
- Both workers and firms have a common discount factor, β and are risk neutral

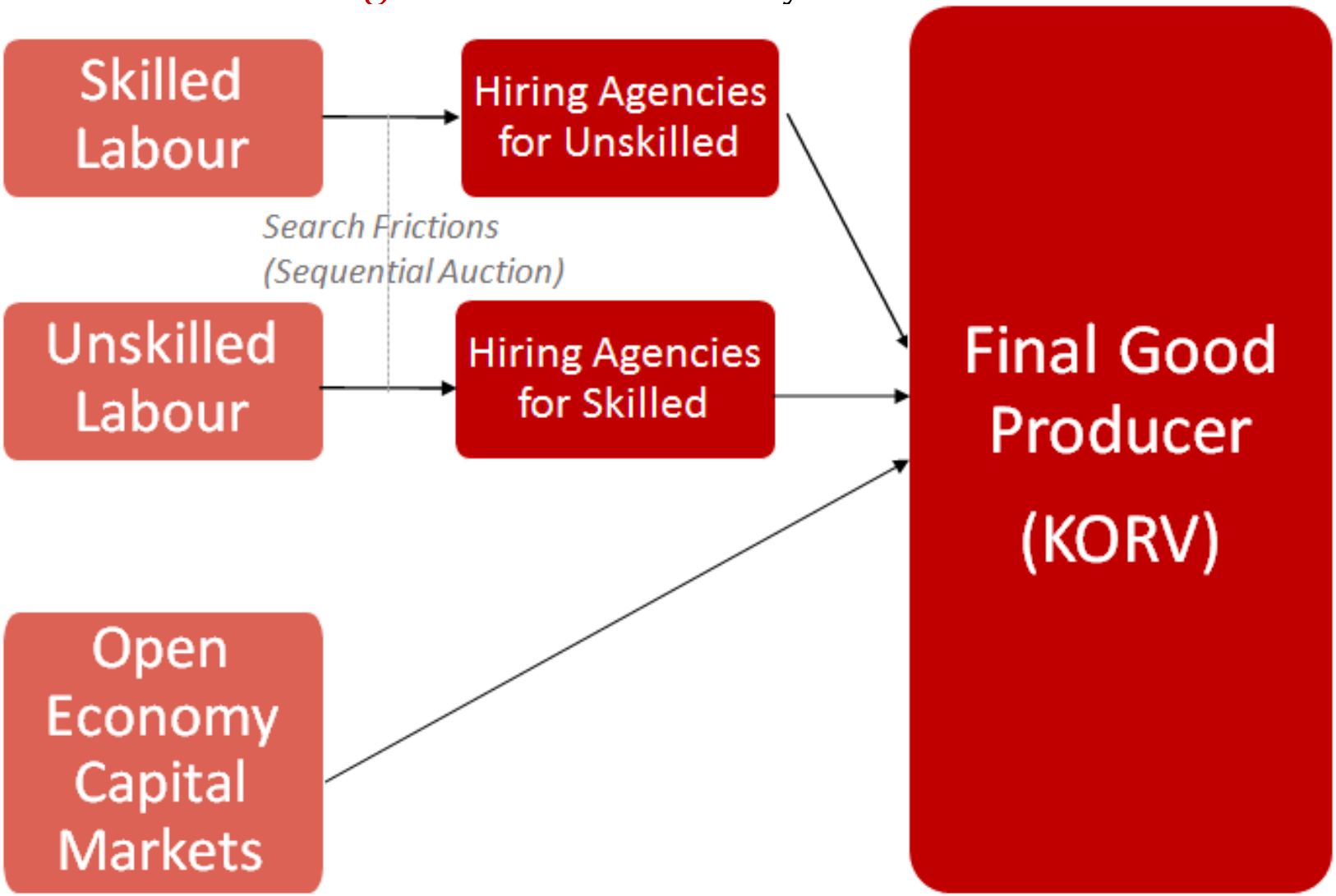
The Model: Environment

Firms

- I wish to allow for both capital to labour substitution in production and substitution between skill types.
- Not easy in pure search/match framework.
- Proposed solution is to have two sectors of production:
  1. **Intermediate sector with search frictions**. Intermediate firms hire labour and sell it onto a final good producer - think of hiring agencies.
     - One segmented intermediate sector for each skill and ability level of workers.
  2. **Final good sector** that combines labour hired in intermediate sector and capital, with no frictions. Capital-skill complementarity as per Krusell et al (2000) - "KORV" production function.

The Model: Environment

Figure: Model Economy Overview



The Model: Environment

Final Good Firms

- Competitive firms which produce using technology shown below.Inputs used:
  - K_{eq} is amount of capital equipment, K_{st} is amount of capital structures
  - U is effective amount of goods purchased from the low skill intermediate sectors, S is total effective labour from high skill intermediate sectors

$$Y = AK_{st}^{\alpha}[\mu U^{\sigma} + (1-\mu)(\lambda K_{eq}^{\rho} + (1-\lambda)S^{\rho})^{\frac{\sigma}{\rho}}]^{\frac{1-\alpha}{\sigma}} \tag{1}$$

$$U = \left(\sum_{i=1}^{M}(x_{i,u}y_{i,u})^{\frac{\Psi_u - 1}{\Psi_u}}\right)^{\frac{\Psi_u}{\Psi_u - 1}}, S = \left(\sum_{i=1}^{M}(x_{i,s}y_{i,s})^{\frac{\Psi_s - 1}{\Psi_s}}\right)^{\frac{\Psi_s}{\Psi_s - 1}} \tag{2}$$

The Model: Environment

Intermediate Firms

Notation:j will be a vector valued index containing both the broad skill index, $h \in \{u, s\}$, and ability index, $i \in \{1..M\}$, of a worker e.g. $j \equiv (h, i)$.

- One intermediate sector for each worker type j.
- One intermediate firm for every worker (so density of intermediate firms = density of workers)
- Number of matches given by matching function $M(S_j, V_j)$. $S_j =$ number of effective type j job searchers. $V_j =$ vacancies.
- $\theta_j \equiv V_j / S_j$ denotes labour market tightness
- Contact rate for type j firms is $q(\theta_j) \equiv M(S_j, V_j) / V_j$, and $(\theta_j q(\theta_j)$, $\chi\theta_j q(\theta_j))$ are the contact rates for unemployed and employed workers respectively.
- Vacancies determined by free entry : i.e. firms issue a vacancy until expected profit equals vacancy cost.

The Model: Environment

Intermediate Firms: Wage Setting

- Assume that firms and unemployed workers engage in Nash bargaining - the minimum wage acts as a constraint to the Nash maximisation.
- When workers gets poached, incumbent and rival bid-up the wage until the value of employing a poached worker equals the value of carrying a vacancy i.e. zero (Postel-Vinay and Robin (2002))
- Therefore poached workers will get paid their marginal product in final good production.
- **Minimum Wage reduces expected profit from employing not-poached worker, and decreases vacancy creation**

The Model: Behaviour

Workers

A worker of a given type j exist in one of three states:

- unemployed, receiving flow income b,with lifetime value function denoted V_j^{ue}
- employed but not poached, receiving the higher of Nash bargained wage w_j^b and the minimum wage m_w, with value function V_j^{np};
- employed and poached, receiving wage w_j^p, with value function V_j^p

$$V_j^{ue} = b + \beta[\theta_j q(\theta_j) V_j^{np} + (1 - \theta_j q(\theta_j)) V_j^u] \tag{3}$$

$$V_j^{np} = \max(w_j^b, m_w) + \beta\left[\delta_j V_j^{ue} + (1-\delta_j)[\chi\theta_j q(\theta_j) V_j^p + (1 - \chi\theta_j q(\theta_j)) V_j^{np}]\right] \tag{4}$$

$$V_j^p = w_j^p + \beta[\delta_j V_j^{ue} + (1-\delta_j) V_j^p] \tag{5}$$

The Model: Behaviour

Final Good Producers

- The firm's profit maximisation problem is:

$$\max_{K_{st},K_{eq},h_{i,u},h_{i,s}\forall i\in 1..M} \Pi = AK_{st}^{\alpha}\left[\mu U^{\sigma} + (1-\mu)(\lambda K_{eq}^{\rho} + (1-\lambda)S^{\rho})^{\frac{\sigma}{\rho}}\right]^{\frac{1-\alpha}{\sigma}}$$

$$-\sum_{i=1}^{M} p_{i,u}h_{i,u} - \sum_{i=1}^{M} p_{i,s}h_{i,s} - r_{st}K_{st} - r_{eq}K_{eq} \tag{6}$$

$$U = \left(\sum_{i=1}^{M}(x_{i,u}h_{i,u})^{\frac{\Psi_u - 1}{\Psi_u}}\right)^{\frac{\Psi_u}{\Psi_u - 1}}, S = \left(\sum_{i=1}^{M}(x_{i,s}h_{i,s})^{\frac{\Psi_s - 1}{\Psi_s}}\right)^{\frac{\Psi_s}{\Psi_s - 1}}$$

- Since final good producer is assumed to be competitive, all inputs are chosen to equalise marginal product is with the price of input.

The Model: Behaviour

Intermediate Firms

- Exist in one of three states:
  - carrying a vacancy, with firm value denoted by J_j^v ,
  - employing a not-poached worker, J_j^{np}, and
  - employing a poached worker, with value J_j^p.

$$J_j^v = -\kappa_j + \beta[q(\theta_j)\{s_j^u J_j^{np} + (1 - s_j^u) J_j^p\} + (1 - q(\theta_j)) J_j^v]$$

$$J_j^{np} = p_j - \max(w_j^b, m_w) + \beta\left[(1 - \delta_j)\{\chi\theta_j q(\theta_j) J_j^p + (1 - \chi\theta_j q(\theta_j)) J_j^{np}\} + \delta_j J_j^v\right]$$

$$J_j^p = p_j - w_j^p + \beta[(1 - \delta_j) J_j^p + \delta_j J_j^v]$$

The Model: Behaviour

Intermediate Firms

- Free entry, so $J_j^v = 0$, and Betrand competition between employers implies $J_j^p = 0$ so $w_j^p = p_j$.
- From these we get no entry condition:

$$\kappa_j = \beta q(\theta_j) s_j^u \frac{p_j - \max(w_j^b, m_w)}{1 - \beta(1-\delta_j)(1-\chi\theta_j q(\theta_j))} \tag{10}$$

- The bargained wage is given below (Φ is the nash bargaining parameter):

$$\begin{aligned} w_j^b &= \underset{w_j^b}{\operatorname{argmax}} (V_j^{np} - V_j^u)^{\Phi_j} (J_j^{np})^{1-\Phi_j} \\ &= \Phi_j p_j + (1-\Phi_j)\left(V_j^u(1-\beta) - \beta(1-\delta_j)\chi\theta_j q(\theta_j)(V_j^p - V_j^u)\right) \end{aligned} \tag{11}$$

The Model: Equilibrium

Equilibrium: a sketch

- Steady State in Labour Markets

$$\delta_j(1 - e_j^{ue}) = \theta_j q(\theta_j) e_j^{ue} \tag{12}$$

$$(\delta_j + \chi\theta_j q(\theta_j)) e_j^{np} = \theta_j q(\theta_j) e_j^{ue} \tag{13}$$

- Solving gives us steady state unemployment and labour market tightness: $e_j^{ue^{ss}}, \theta_j^{ss}$
- Intermediate goods market clearing:

$$p_j^s = \max(w_j^b, m_w) + \frac{\kappa_j \left(1 - (\beta(1 - \delta_j)(1 - \chi\theta_j^{ss} q(\theta_j^{ss})))\right)}{\beta q(\theta_j^{ss}) s_j^u} \tag{14}$$

$$p_j^d = \frac{\partial Y}{\partial h_j(e_j^{ue^{ss}})} \tag{15}$$

The Model: Minimum Wage Impacts

- From equilibrium conditions:

$$\max(w_j^b, m_w) = \frac{\partial Y}{\partial h_j(e_j^{ue^{ss}})} - \frac{\kappa_j \left(1 - (\beta(1-\delta_j)(1-\chi\theta_j^{ss} q(\theta_j^{ss})))\right)}{\beta q(\theta_j^{ss}) s_j^u} \quad (16)$$

- So wages = **marginal product of labour** minus **recruitment costs**
- Minimum wage increase implies:
  - intermediate firms to decrease vacancies. CD matching function: probability of filling remaining vacancies increaes **reducing recruitment cost**.
  - reducing vacancies decreases employment, increasing **marginal product of labour**.

Calibration Approach

- Standard(ish) macro story: borrow some parameters from literature, estimate others (by SMM).
- I focus on estimating parameters for:
  1. exogenous distributions of worker ability (log normal), with seperate distributions for unskilled and skilled.
     - *Empirical Targets: Variance of Log Wages and p90-10 ratios*
  2. the elasticities of substitution between workers within these two skill classes, ψ_u, ψ_s,
     - *Empirical Targets: Unemployment Rates: levels (2013) and changes (2013-17)*
  3. recruitment costs κ_u, κ_s
     - *Empirical Targets: Unemployment Rates*
  4. the share parameter, μ, in the KORV production function.
     - *Empirical Targets: Graduate Wage Premium*

Calibrated Parameters

Table: Estimation Results: Method Comparison

Moment	Model Moment	Empirical Moment
Median Hourly Wage: Unskilled	10.03	9.5
Median Hourly Wage: Skilled	15.77	15.71
Var Log Wages: Unskilled	0.47	0.49
Var Log Wages: Skilled	0.53	0.57
p90/50 Wages: Unskilled	2.0	1.92
p90/50 Wages: Skilled	2.01	1.96
p50/10 Wages: Unskilled	1.59	1.57
p50/10 Wages: Skilled	2.1	2.07
Min Wage Coverage: Unskilled	0.17	0.16
Min Wage Coverage: Skilled	0.06	0.06
Unemployment: Unskilled	0.07	0.07
Unemployment: Skilled	0.03	0.03
Δ Unemployment Unskilled (2013-17)	0.01	0.0
Δ Unemployment Skilled (2013-17)	0.0	0.0

Calibrated Parameters

Table: Estimated Parameters

Parameter	Description	Value
Ψ_u	Elasticity of substitution between unskilled workers	3.611
Ψ_s	Elasticity of substitution between skilled workers	8.889
μ	Share parameter determining skill premium in KORV production function	0.299
A	Total Factor Productivity	7.581
η_u	Variance parameter of worker ability distribution: unskilled workers	0.343
η_s	Variance parameter of worker ability distribution: skilled workers	0.419
ϕ_u	Nash Bargaining Parameter for unskilled workers	0.264
ϕ_s	Nash Bargaining Parameter for skilled workers	0.146
κ_u	Hiring cost: unskilled workers	3425.825
κ_s	Hiring cost: skilled workers	4116.545

Calibrated Parameters

Table: Calibrated Parameters

Parameter	Description	Source	Value
δ_u	Job destruction rate: unskilled	LFS 2013q4-2014q3	0.011
δ_s	Job destruction rate: skilled	LFS 2013q4-2014q3	0.007
χ_u	Relative search intensity of employed to unemployed: unskilled	LFS 2013q4-2014q3 (ratio of employer change rate to unemployment exit)	0.112
χ_s	Relative search intensity of employed to unemployed: unskilled	LFS 2013q4-2014q3 (ratio of employer change rate to unemployment exit)	0.075
b	Monthly Unemployment benefits (job seekers allowance)	Legislative level 2013-14	313.492
m_w	Hourly minimum wage	Legislative level 2013-14	6.31
σ	Elasticity of substitution between unskilled and skilled workers	Krusell et al. (2000)	0.401
ρ	Elasticity of substitution between skilled workers and capital equipment	Krusell et al. (2000)	-0.495
α	Capital Structures Parameter	Krusell et al. (2000)	0.117
λ	Input share parameter for capital equipment and skilled labour	Krusell et al. (2000)	0.3
γ	Matching Parameter	Hagedorn and Manovskii (2008)	0.407
β	Monthly discount factor for workers and firms	By assumption	0.996

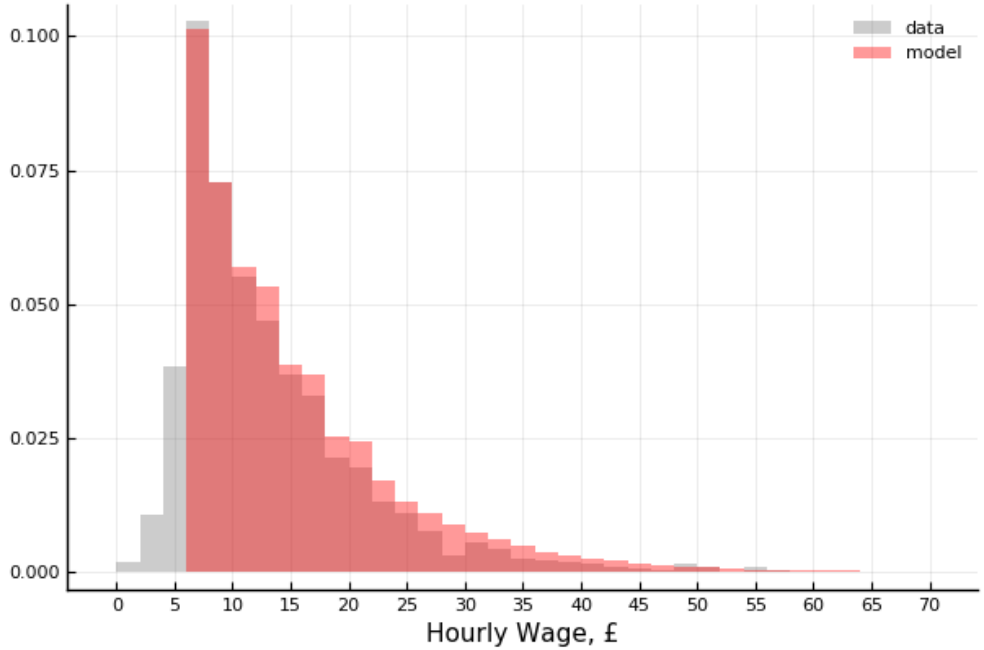
Out of Sample Performance

- Model is under-predicting level of mark-ups in economy (OTJ search impacts too stark?)

Table: Non-targeted Macro Moments

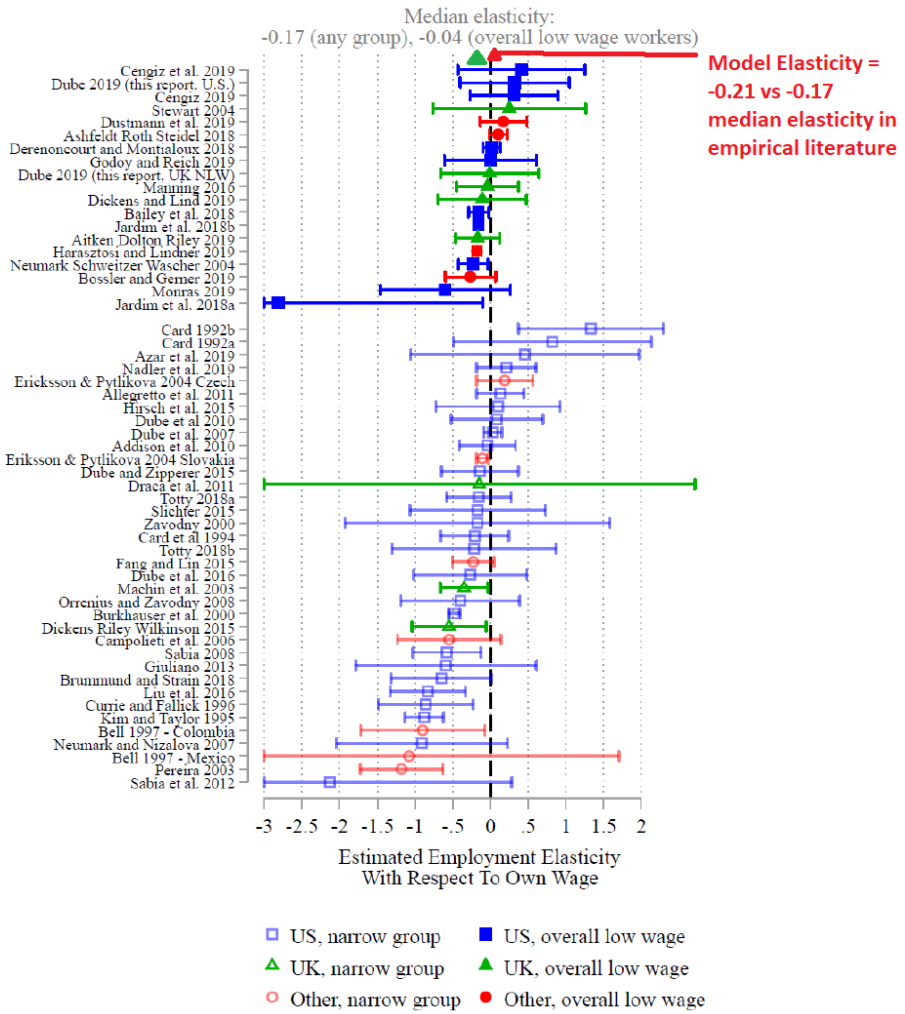
Moment	Model Moment	Empirical Moment
Labour Share of GVA[1]	0.76	0.76
Mark-Up Ratio[2]	1.06	1.5
Net Capital Stock/GVA[3]	1.77	2.6

[1] Bank of England, includes self-employed labour income (imputing it as compensation per employee multiplied by number of self-employed).
[2] Empirical moment taken from De Loecker and Eeckhout (2018), model moment is calculated analagously (as described in text).
[3] UK National accounts, ONS.



Results: Matching Reduced Form Evidence

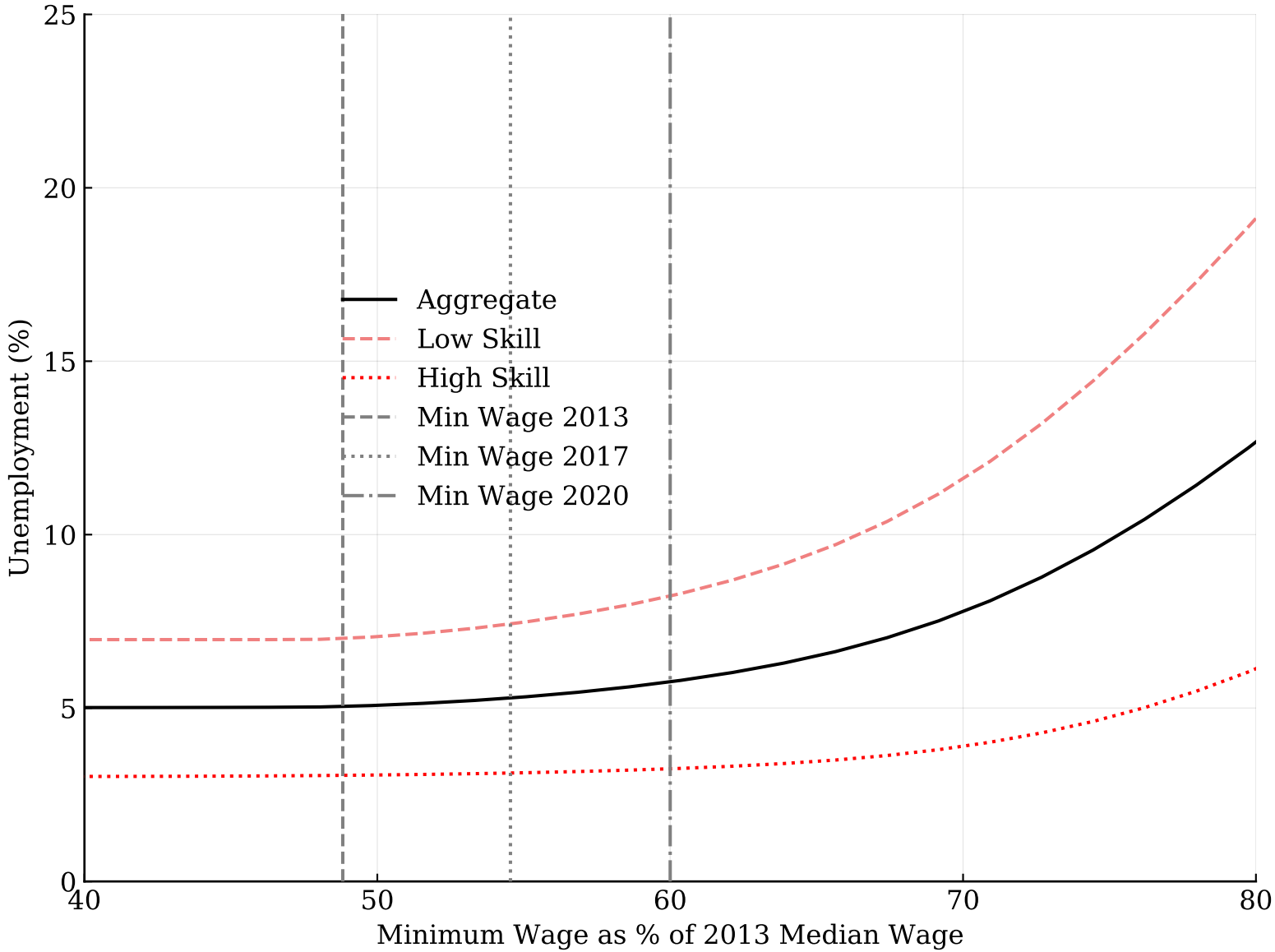
Figure: Model vs Empirical Evidence



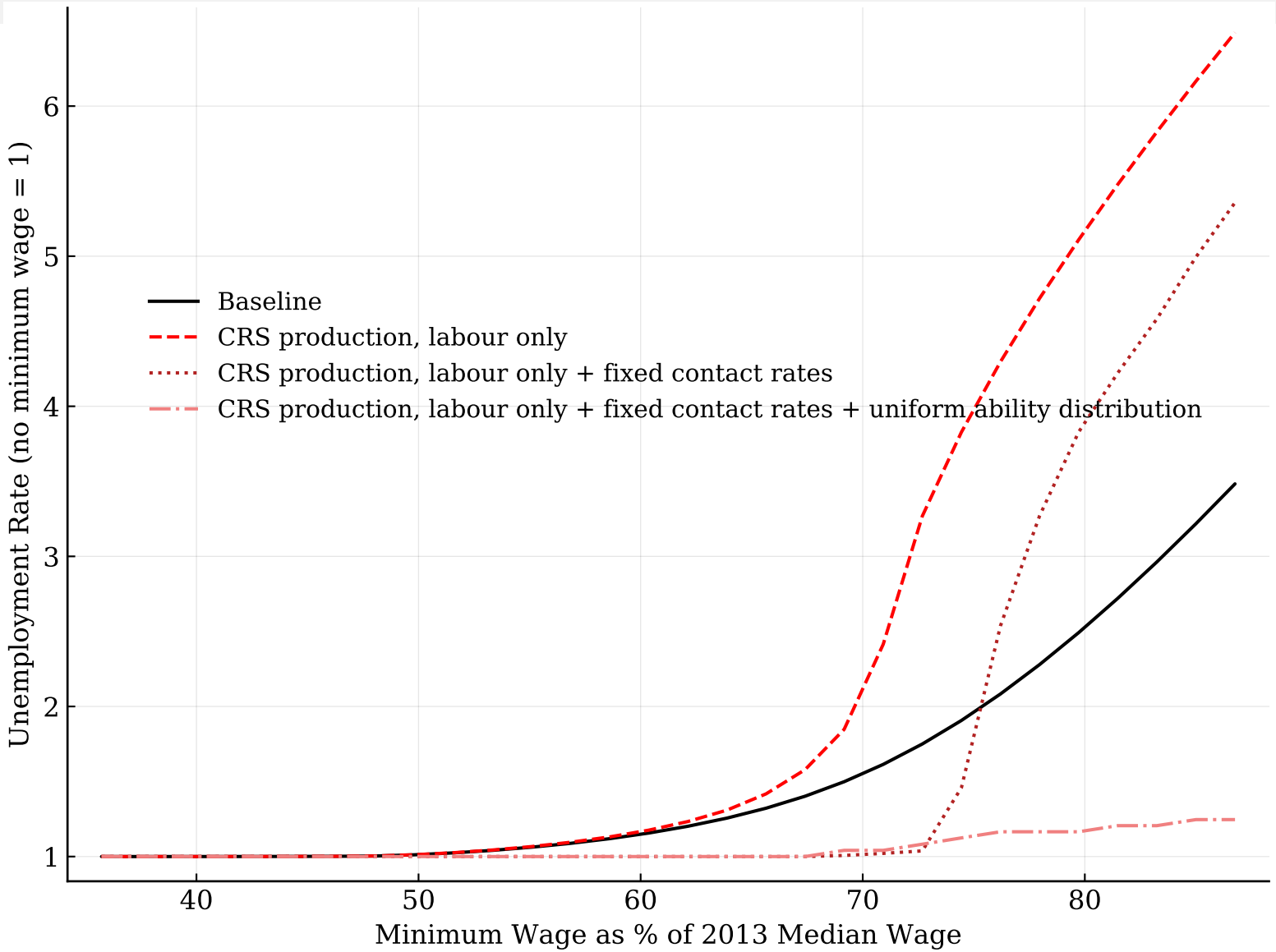
- Model hits empirical employment elasticity from UK minimum wage introduction (almost by design), but significantly under predicts impact on wages and firm profits.
- Above finding suggests might be worth introducing firm heterogeneity

Results: Searching for Nonlinearities

Figure: Unemployment Response



Results: Drivers of Nonlinearities



Conclusions and Next Steps

- I develop a model that combines search frictions with a production function featuring several margins of substitution between factor inputs.
- Nonlinear unemployment reaction in model from non-uniform distribution of skills; endogenous vacancy creation, imperfect substitution between factor inputs.
- When calibrated to the UK economy, we find:
  1. quantitatively, imperfect substitution between inputs is an important endogenous source of nonlinearities.
  2. nonlinear unemployment reaction of unskilled workers starts to bite (gently) when minimum wage is around 55-60% of median wage.
- Next Steps: Introduce firm heterogeneity? Might help model match both wage, profit and unemployment reaction to minimum wage increases. Also model could address emerging evidence from Germany.